Size and scale effects in composites: II. Unidirectional laminates

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Abstract

This paper is concerned with the scale and size effects in the strength characterisation of composite materials with particular reference to hand laid-up unidirectional laminates. Attention has been focused on the tensile and flexural strengths of glass/epoxy and carbon/epoxy laminates. The experiments have been designed through the use of statistical, factorial-based techniques, whereby a more comprehensive analysis of the inter-relationship between different parameters and their influence on strength can be made. The work highlights the importance of fabrication factors and the distinguishing difference between scale effects and size effects.

Keywords

Scale effects, Size effects, FRP, Composites, Dimensional analysis, Weibull analysis, Factorial experiments

1. Introduction

The relatively recent introduction of FRP composite technology, together with the large range of materials used and being introduced, mean that a broad design base, such as that available for many metals, has not yet been compiled for FRP materials. Hence much testing of composite components has to be carried out either on full-scale prototypes, or, in order to save both time and expense, on small-scale models by use of the principles of dimensional analysis. It follows therefore, that any discrepancies encountered whilst scaling from model to full size (i.e. any size effects) should be both identified and understood. Similarly, much of the design of composite components is based on material properties derived from small laboratory-scale coupons, often leading to a trial and error approach if the properties obtained in the laboratory tests do not correctly predict the component behaviour.

It has been thought for some time that a strength `size effect' may exist for some composites, which is usually (but not exclusively) detrimental with increasing size. This is thought to be due to the increased probability of a larger specimen containing a flaw large enough to lead to failure. However, an accurate quantitative description of such effects, or even concrete evidence of their existence, has proved elusive. In a review of this literature, see Sutherland et al. [1], it was shown that the majority of the existing work in the field concerns high quality, pre-preg carbon/epoxy laminates for use in the aerospace industries. It was demonstrated that a number of authors have come to the conclusion that the scale/size phenomenon exists and that statistical strength theory may be used to quantify it. However, despite the fact that this theory is statistical in nature, very few statistical analyses of the results and trends are reported.

The scaling problem is especially complex for composites on account of the intricate nature of their micro-structure. Also the many possible material properties that may be considered, such as manufacturing technique and conditions, and fibre and matrix materials, further complicate the problem. Further still, the mechanical testing of composite materials is a contentious subject, with many variables that may affect the observed material properties. The marine industry generally uses more variable, hand laid-up laminates than the aerospace industry, leading to greater variation in mechanical properties.

Thus an investigation of composite materials strength size effects concerns both a number of pertinent variables, and also experimental data subject to considerable scatter. This type of problem requires an efficient experimental programme and statistical analysis techniques in order to separately estimate the effects of each variable, and also to distinguish these effects from the random variation in the experimental data. The methods of statistically designed experimentation have been developed to benefit exactly this type of problem, and hence it is advantageous to use them here. This represents a new approach to the question of strength size effects for composite materials, and indeed to the testing of composites in general.

Hence, the aims of the overall study were two-fold; to investigate the size effects in composite materials, and also to introduce the reader to the basics of experimental design techniques, showing the application of these methods as applied to composites testing. The full test programme also involved an extensive study into size effects in woven roving glass/polyester shipbuilding quality composites, (see Refs. [1, 2]). The purpose of this paper is to report on the complex interactions among the variables affecting unidirectional laminate tensile and flexural strengths by using statistical experimental design techniques.

2. Factorial experiments

2.1. Overview

Experimental design has been defined as the `purposeful changes of the inputs (factors) to a process in order to observe the corresponding changes in the outputs (responses)' [3]. An introduction to the subject is given in this paper with particular emphasis on concepts and techniques employed in the experimental work described in Section 3. More detailed and wider-ranging information is available in the literature, for example, [4, 5].

A characteristic of many statistically designed experiments is that they investigate the joint influence of more than one variable on the performance of a system. This is achieved by changing the values of the variables simultaneously. This approach is better than the alternative of looking at each variable individually in a series of separate experiments since a more complete understanding of the features of the system and how they interact may be gained. The basic terms used in experimental design will now be defined in order to simplify further descriptions of this subject.

Response variable: This is the output or measurement of interest from the system under consideration, for example the strength of a composite coupon.

Factors: Factors are those features of the system which are controllable in the experiment and whose influence on the response is of interest to the experimenter, for example specimen size and fibre reinforcement material. Factors may fall into one of two groups;

1. *Quantitative* factors whose values can be arranged in order of increasing magnitude, for example length,
2. *Qualitative* factors whose values cannot be arranged in order of increasing magnitude, for example fibre reinforcement material.

Factor levels: The values taken by a factor in an experiment are called the levels of the factor. The experimenters, together with as many persons familiar with the system as possible, should set the factor levels on the basis of existing knowledge to encompass the regions of operation for investigation. Most designed experiments use only 2 or 3, or occasionally as many as 4 or 5, levels for their factors in order to keep the size of the experiment practicable.

Treatment combination: A treatment combination is a specified combination of factor levels at which an experimental run is made in order to make an observation.

Full factorial experiment: In such an experiment, a response is measured at every possible treatment combination.

Fractional factorial experiment: Here only a subset of all possible treatment combinations is used in the experiment.

Effects: The change in the response variable caused by changing the level taken by a factor is referred to as the *main effect* of the factor. Any variation of the effect of a factor with the level taken by another factor is described by an *interaction effect*. Both of these terms will be explained further in Section 2.2.

2.2. Factorial experimentation

A common method of scientific experimentation entails keeping all factors constant at a `base' level for the first trial, using this response as a reference value, and then varying each factor individually, i.e. a `one-factor-at-a-time' experimental design. The incentive for this approach is a desire to carry out as few trials as possible, which on face value seems to be satisfied by this design. Also, practitioners will often claim that this type of experiment gives results which are “easier to understand”. However this is a false logic for one-factor-at-a-time testing is seriously flawed, as will be shown in this section. A far more efficient method is that of factorial experimentation, where several factors are changed before each run. The changes are not made in an arbitrary manner, but in such a way that the overall plan enables the effects of each factor to be retrieved from the data. In fact, not only the experimental plan, but also the whole approach to statistically designed experiments should be systematic rather than *ad hoc* so that the data can be interpreted sensibly.

To illustrate the increased efficiency obtained through the use of a factorial experiment consider the simple case where there are three factors each at two levels. The full factorial two-level, three-factor experimental plan consists of eight runs, and hence it is possible to examine seven comparisons in the data. These may be chosen to be the estimates of the main and interaction effects, as follows. It is convenient to represent the lower level of each factor (`lower' will be arbitrary for a qualitative factor) with a `−' and the upper level with a `+'. The full factorial design is shown in Table 1 and the equivalent `one-at-a-time' experiment in Table 2, where each row represents a treatment combination or experimental trial, for example a test on an individual specimen. The meaning of the word `equivalent' as used here will become apparent.

Run Number	Factor A	Factor B	Factor C
1	-	-	-
2	+	-	-
3	-	+	-
4	+	+	-
5	-	-	+
6	+	-	+
7	-	+	+
8	+	+	+

Table 1. Full Factorial Experimental design

Run Number	Factor A	Factor B	Factor C
1	-	-	-
2	-	-	-
3	-	-	-
4	-	-	-
5	+	-	-
6	+	-	-
7	+	-	-
8	+	-	-
9	-	+	-
10	-	+	-
11	-	+	-
12	-	+	-
13	-	-	+
14	-	-	+
15	-	-	+
16	-	-	+

Table 2. One-factor-at-a-time Experiment

An important property of the array in Table 1 is that of *Orthogonality*; more specifically, the array has a useful pattern for the levels taken by pairs of factors. For example on inspection it can be seen that for the four runs with A at the − level, two are with B at the + level and two are with B at the − level, and the same pattern occurs for the four runs with A at the + level. This pattern is evident when any pair of factors is considered. At this point it is helpful to introduce a simple statistical model for the response which includes only main effects;

$$y = \alpha a + \beta b + \gamma c + Const. + \varepsilon \quad (1)$$

where *a*, *b* and *c* are coded −1 and +1 for low and high levels of factors A, B and C, respectively, α,β and γ are unknown constants and ε is a random error term.

Hence, for the treatments (runs) with A at the low level in Table 1 we have:

$$y_1 = -\alpha - \beta - \gamma + Const. + \varepsilon_1$$
$$y_3 = -\alpha + \beta - \gamma + Const. + \varepsilon_3$$
$$y_5 = -\alpha - \beta + \gamma + Const. + \varepsilon_5$$
$$y_7 = -\alpha + \beta + \gamma + Const. + \varepsilon_7 \quad (2)$$

It can now be seen that the average value for the response with A at the low level is independent of β and γ;

$$y_{A-} = -\alpha + Const. + \varepsilon_{A-} \quad (3)$$

where ε_{A-} denotes the average of the four error terms. Similarly for A at the high level;

$$y_{A+} = +\alpha + Const. + \varepsilon_{A+} \quad (4)$$

The orthogonal factorial design gives three main advantages:

(i) The effect of each factor may be easily separated through an averaging process.

(ii) The effect of altering a factor level is seen at more than one level of the other factors.

(iii) The effects of each factor may be estimated with greater precision.

The first advantage is important as it benefits the statistical precision of the estimates of the parameters from the designed experiment. For example the full factorial design will effectively have four runs available for estimating the response at each level of a factor, since the effects of changing the levels of the other factors between these runs cancel under model (1). Because of this, the full factorial design needs only 8 runs to give 4 replications for each level of each factor, whereas the equivalent one-factor-at-a-time experiment requires 16 trials for the same precision.

The second advantage makes possible an investigation of the joint action of factors on the response through the examination of *interactions*. An interaction between two factors occurs when the effect of changing one factor is not constant but depends on the value taken by the other factor. This is illustrated graphically in Fig. 1.

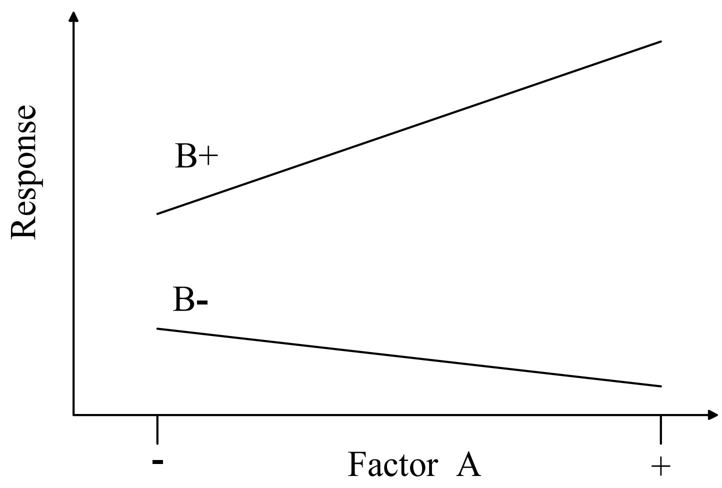

Fig. 1. Illustration of Interaction.

This is only one specific form of interaction; it can also give other lines that are not parallel, which may cross, diverge, or converge. An interaction between two factors A and B is denoted by AB or A × B and is called *a two-way interaction* since it involves two factors. If this interaction is itself dependent upon the level of another factor, C say, then we have what is termed a *three-way interaction*, denoted by ABC or A × B × C. An example of this type of interaction is illustrated in Fig. 2.

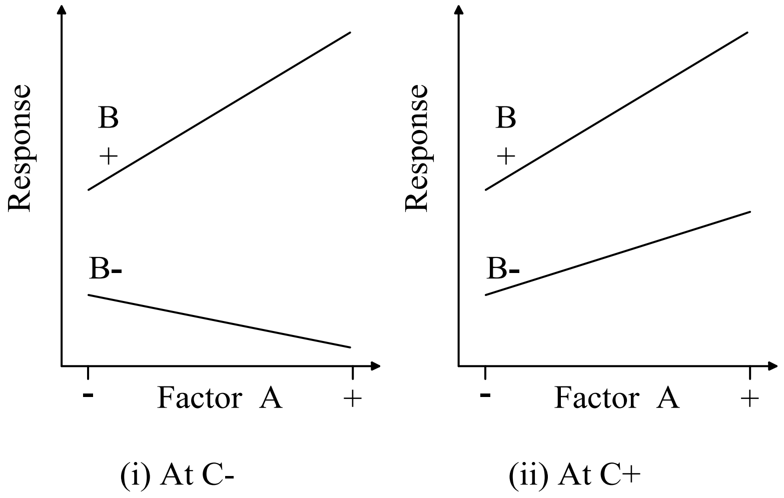

Fig. 2. Illustration of a three-way interaction.

The main effects of factors are meaningless in isolation when there are any important interaction effects. This can be seen when we introduce interactions into the previous statistical model for the response;

$$y = \alpha a + \beta b + \gamma c + \delta ab + \varsigma ac + \eta bc + \lambda abc + Const. + \varepsilon \quad (5)$$

where the products *ab*, *ac* and *bc* represent two-way interactions, *abc* represents a three-way interaction and α,β,…,λ are unknown constants.

The importance of the second advantage of orthogonal designs given above can now be seen; viewing the effect of a factor at differing values of other factors enables any interactions to be estimated. Compare this to the one-factor-at-a-time experiment where, for example `A+' only occurs when B and C are at the − level, rendering it impossible to examine any interaction effects.

If it is known a priori that some or all of the interactions between the factors are negligible then the number of experimental runs needed to give the required information may be reduced through the use of a fractional factorial design. However, this also reduces the precision with which the main effects and any other interactions can be estimated and a compromise through the design of the experiment must be reached. Such a design may be chosen to be orthogonal; a small example is shown in Table 3 where the runs are a subset from the full factorial design of Table 1.

The examples used so far have all concerned only two levels for each factor. However if more information is required on the form of the relationship between a factor and the response then three, four or even five levels may be used. The design and data analysis techniques used for experiments with factors at more than two levels are explained further in Refs. [4, 5].

Run Number	Factor A	Factor B	Factor C
1	-	-	+
2	+	-	-
3	-	+	-
4	+	+	+

Table 3. Fractional factorial design

The nature of the system studied may require a more sophisticated design structure. If the levels taken by one factor must be different for each level of another factor then a *nested* design structure is needed and the former factor is said to be nested under the latter. For example, flexural testing may require the length to depth ratio to be within a certain range and this may lead to lower length values for thinner specimens than for thicker ones. Here length is said to be nested under thickness. In some experiments certain factors have to be used on a large section of experimental material (called *whole-plots*) and other factors pertain to smaller parts (*sub-plots*) within the whole-plots. There may be variations between whole-plots, and different variation between the sub-plots within each whole-plot. An example is the fabrication of composite specimens for mechanical testing whereby specimens of different thicknesses must originate from different panels due to the nature of the fabrication process. Nested and split-plot designs are further explained in the literature, for example, Ref. [6].

2.3. Statistical analysis techniques

Analysis of the results of the experiment aims to determine whether or not an observed effect is due to changes made between treatments (`real') or solely due to the random residual variation from uncontrolled noise factors. Simple statistical methods, especially graphical methods such as normal plots, are advocated for this task by Grove and Davis [4], after which the relevant effects and interactions may be included in an analysis such as linear regression or ANOVA to give a prediction equation. However, especially when one factor effect is much larger than the others, this approach may lead to important factors being overlooked. Hence the approach used in this study is to include all pertinent main effects and two-way interactions in an ANOVA or regression analysis [7] using for example SAS software (SAS Institute Inc., 1989) and then to interpret the forms of the important effects graphically.

3. Experimental programme

3.1. Test procedure

Previous work by Shenoi et al. [8] had developed manufacturing procedures using unidirectional glass/epoxy laminates and so, to build on this experience, the same methods and materials were used here. To provide a link between the mainly aerospace orientated work to the planned future studies of marine quality composites, unidirectional carbon/epoxy was also included. The composites were produced using a vacuum assisted hand lay-up method, to give a normal fibre volume fraction of 40%. The material used were Vetrotex 450 g/m^2 unidirectional E-glass, S.P. Systems 300 g/m^2 unidirectional carbon

(UC300/350A), supplied in the form of tapes consisting of bundles of fibre 2–3 mm and 4–5 mm wide, respectively, and S.P. Systems Ampreg 20 epoxy with standard hardener.

The tensile response and strength of composite materials is important from a design point of view, and an easy, economical and often used method is that of flexural testing. Hence, both tensile and flexural tests were carried out using an Instron 3664 at constant cross-head speed.

A load length of one third the support length was used for the flexural tests, and the load and support rollers were clamped in place in order to avoid slippage. An even distribution of the load was ensured by allowing the loading block, and hence the two load rollers, to pivot with respect to the cross-head. To avoid damage due to the load rollers, especially for the smaller specimens, the strain gauges were fitted in the centre of the lower (i.e. tensile) face. The tensile specimens were secured at each end using self-tightening `wedge' type grips, one of which was attached to the rig via a universal joint. The strain gauges were attached to the centre of one face. A simple rectangular specimen geometry was used consistent with established practice in the regulations of the marine industry (Lloyd's Register of Shipping [9]).

For both types of test, the width and depth of each specimen was measured at five position along the length using a micrometer before commencing the test. Any significant features of the specimen such as voids were also recorded at this stage. After starting the test, the load at which the first cracks or clicks were heard was noted and then the failure sequence was described quantitatively. Analogue plots of load from the Instron load cell versus cross-head deflection were obtained for each test. Graphs of load versus strain from the strain gauges were also produced.

3.2. Experimental design

Various authors have postulated that any size effect may depend upon the way in which the volume of the composite material is increased. Hence the length, width and thickness of the test specimens were varied separately. In order to allow greater variation between the levels of width for the tensile tests, and length for the flexural tests, a nested structure of length and width under thickness (number of plies) was formulated. Width and length were varied within each of three thicknesses. For the flexural tests the influence of the diameter of the loading and support rollers was also investigated.

The lists of factors for the two test methods are given below.

Tensile:

Thickness
Reinforcement
Length
Width

Flexural:

Thickness
Reinforcement
Length
Width
Roller Diameter

Since the series of experiments was to be an introduction to the methods of experimental design, a simple two level full factorial design for the three factor tensile tests was adopted. A single replicate of each specimen led to 2×2^3=16 runs (specimens) at each thickness, i.e. a total of 48 specimens. The flexural tests included an extra factor, namely roller diameter. To achieve the same number of specimens as the tensile tests, and also to broaden the experience with experimental design methods, a half-replicate design was selected for these tests. The coded designs are shown in Table 4 and Table 5 and a schematic of the structure of the unidirectional test programme is given in Fig. 3.

Specimen	**Reinforcement**	**Length**	**Width**
1	Glass	Low	Low
2	Glass	Low	High
3	Glass	High	Low
4	Glass	High	High
5	Carbon	Low	Low
6	Carbon	Low	High
7	Carbon	High	Low
8	Carbon	High	High

Table 4. Experimental design of unidirectional tensile tests

Specimen	**Reinforcement**	**Length**	**Width**	**Roller ∅**
1	Glass	Low	Low	Low
2	Glass	Low	High	High
3	Glass	High	Low	High
4	Glass	High	High	Low
5	Carbon	Low	Low	High
6	Carbon	Low	High	Low
7	Carbon	High	Low	Low
8	Carbon	High	High	High

Table 5. Experimental design of unidirectional flexural tests

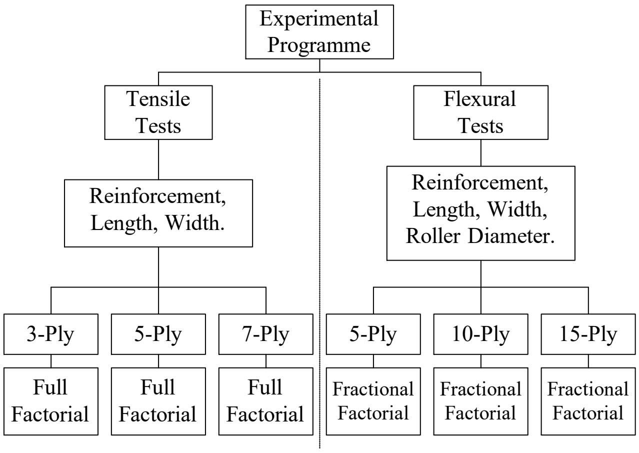

Fig. 3. Schematic of unidirectional test programme.

Each group of 16 specimens of the same thickness were scaled geometrically across thickness, for example the values of the low length and widths of the 10-ply flexural specimens were twice those for the 5-ply specimens. The specimen dimensions were set using the appropriate ASTM standards [10, 11] and are given in Table 6 and Table 7.

Units: mm	Depth	Level	Length	Width	Roller ∅
5 Ply	Glass: 2.25 / Carbon: 2.05	**Low**	54	12.5	6
		High	75	25	12
10 Ply	Glass: 4.5 / Carbon: 4.1	**Low**	108	25	6
		High	150	50	12
15 Ply	Glass: 6.75 / Carbon: 6.15	**Low**	162	37.5	6
		High	225	75	12

Table 6. Unidirectional flexural test factor levels (mm)

Units: mm	Depth	Gripped Length	Level	Length	Width
3 Ply	Glass: 1.35 /	25	**Low**	52.5	3.75
	Carbon: 1.25		**High**	105	7.5
5 Ply	Glass: 2.25 /	40	**Low**	87.5	6.25
	Carbon: 2.05		**High**	175	12.5
7 Ply	Glass: 3.15 /	56	**Low**	122.5	8.75
	Carbon: 2.90		**High**	245	17.5

Table 7. Unidirectional tensile test factor levels (mm)

3.3. Results

The full set of experimental data may be found in Appendix A. The failure of the tensile specimens was thought to originate in the area of the jaws, with varying degrees of secondary shear failure along the specimen length. Failure was generally more catastrophic for the carbon coupons. The flexural glass specimens failed in compression with buckling of the fibres under the load roller, and this was sometimes followed by secondary tensile fibre failure. Failure of the carbon specimens was catastrophic and compressive under the load roller. Further details are given in Ref. [12].

4. Statistical analysis

The GLM procedure of the software package SAS [13] was employed to fit a statistical model, carry out an analysis of variance and to check the validity of the model by graphical analysis of the residuals, (see Ref. [7]).

Two independent comparisons can be made between the three levels of thickness used: a linear term and a quadratic term. The linear effect is the difference between the average response at the low factor level and that at the high factor level. The quadratic effect is the difference between the average response at the medium factor level and the average response at the other two levels. Thus the quadratic effect can be thought of as an indication of the `curvature' in the relationship between the response and the factor level. The `SAS' software was used to further subdivide the main effect of the thickness and its interactions into linear and quadratic components.

The statistical models postulated for both tensile stress and tensile strain may be simply represented by

$$Strength = \alpha_0 + \alpha_1 T_l + \alpha_2 T_q + \alpha_3 R + \alpha_4 T_l xR + \alpha_5 T_q xR + \mathrm{E} + \alpha_6 CL + \alpha_7 CW + \alpha_8 CLxCW + \alpha_9 RxCL + \alpha_{10} RxCW + \varepsilon \quad (6)$$

where the α's are unknown coefficients T_l and T_q represent linear and quadratic components of thickness, R is the reinforcement variable (0 for glass; 1 for carbon), E the whole-plot error, CL and CW are the coded length and width (within the appropriate thickness) and ε the sub-plot error.

Both types of error are assumed to be normally distributed with mean zero and constant variance, and to be independent of each other.

Similarly, for the flexural tests

$$Strength = \alpha_0 + \alpha_1 T_l + \alpha_2 T_q + \alpha_3 R + \alpha_4 T_l xR + \alpha_5 T_q xR + \mathrm{E} + \alpha_6 CL + \alpha_7 CW + \alpha_8 RD + \alpha_9 CLxCW + \alpha_{10} CLxRD + \alpha_{11} CWxRD + \alpha_{12} T_{l+q} xRD + \alpha_{13} RxCL + \alpha_{14} RxCW + \alpha_{15} RxRD + \varepsilon \quad (7)$$

where RD is the load/support roller diameter.

4.1. Findings of the data analyses

The reinforcement material has a very large effect on all but one of the responses considered. To enable a comparison of the whole-plot and sub-plot variation without the overwhelming influence of the reinforcement material, the reinforcement sums of squares values can be removed from the whole-plot totals. Similarly, to compare only material variations, as opposed to testing variations, the roller diameter values can be removed from the flexural test sub-plots totals, see Table 8.

	Whole-Plot SS (%)	**Sub-Plot SS (%)**
Tensile Stress	3.4	1.6
Tensile Strain	0.1	1.0
Flexural Stress	39.1	25.5
Flexural Strain	7.6	0.9

Table 8. “Modified” whole-plot and sub-plot sums of squares

The sums of squares normalised with respect to the sub-plot error are given in Table 9 as an indication of relative importance of the whole-plot factors. Here it must be stressed that these ratios are purely comparative, and that no statistical inferences should be made from them. The important values in the table are underlined.

The trends in the data may be seen in the appropriate main effects and interaction plots in Figures 4 to 11.

Source	Tensile Stress	Tensile Strain	Flexural Stress	Flexural Strain
Thick				
(Linear)	27	1	85	745
(Quadratic)	21	0	6	239
Reinforc	1559	1688	17	12000
Thick*Reinforc				
(Linear)	2	0	2	25
(Quadratic)	5	0	4	0

Table 9. Whole-plot sums of squares normalized with respect to sub-plot error

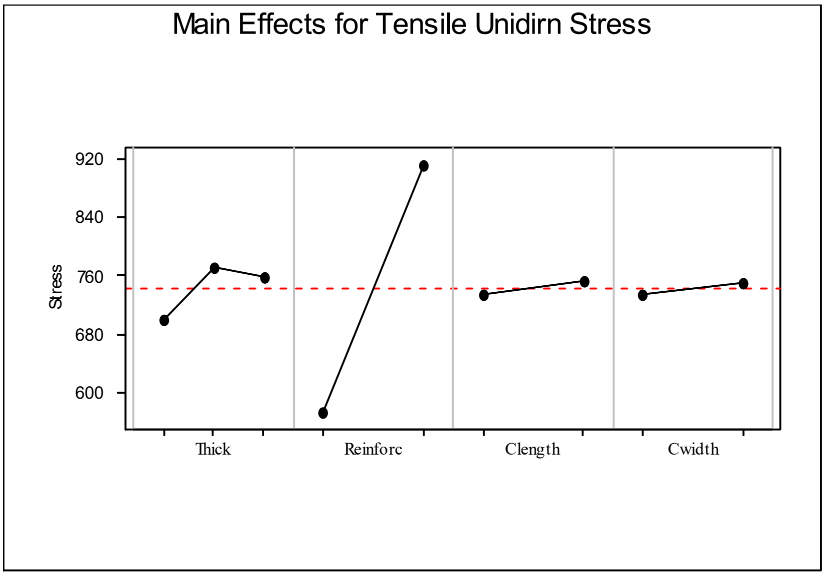

Fig. 4. Tensile stress (MPa) main effects plots.

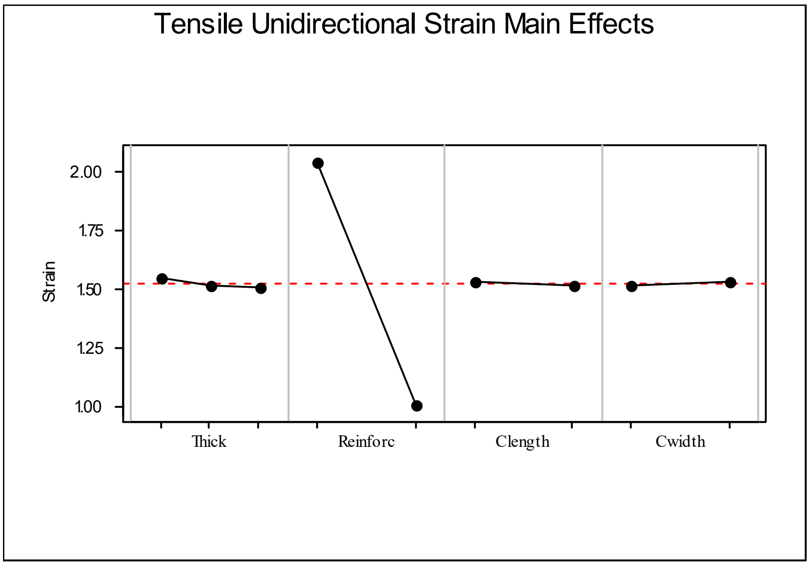

Fig. 5. Tensile strain (%) main effects plots.

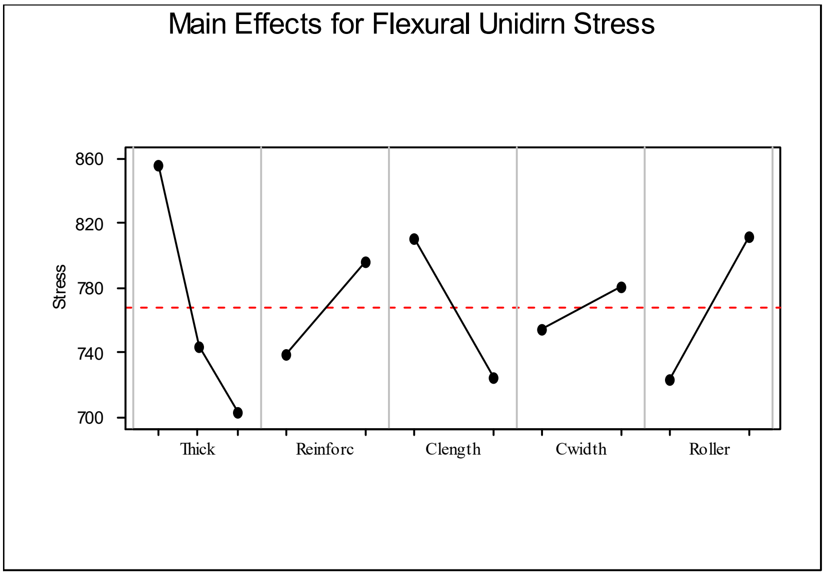

Fig. 6. Flexural stress (MPa) main effects plots.

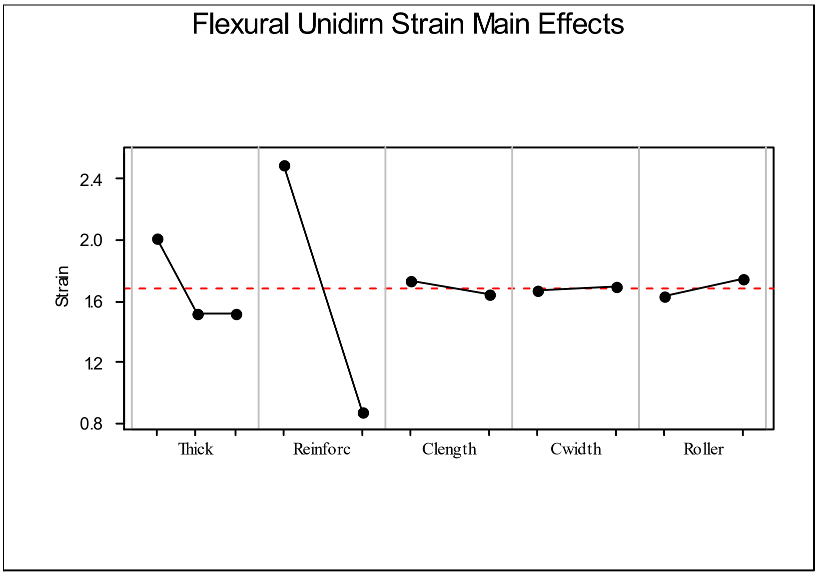

Fig. 7. Flexural strain (%) main effects plots.

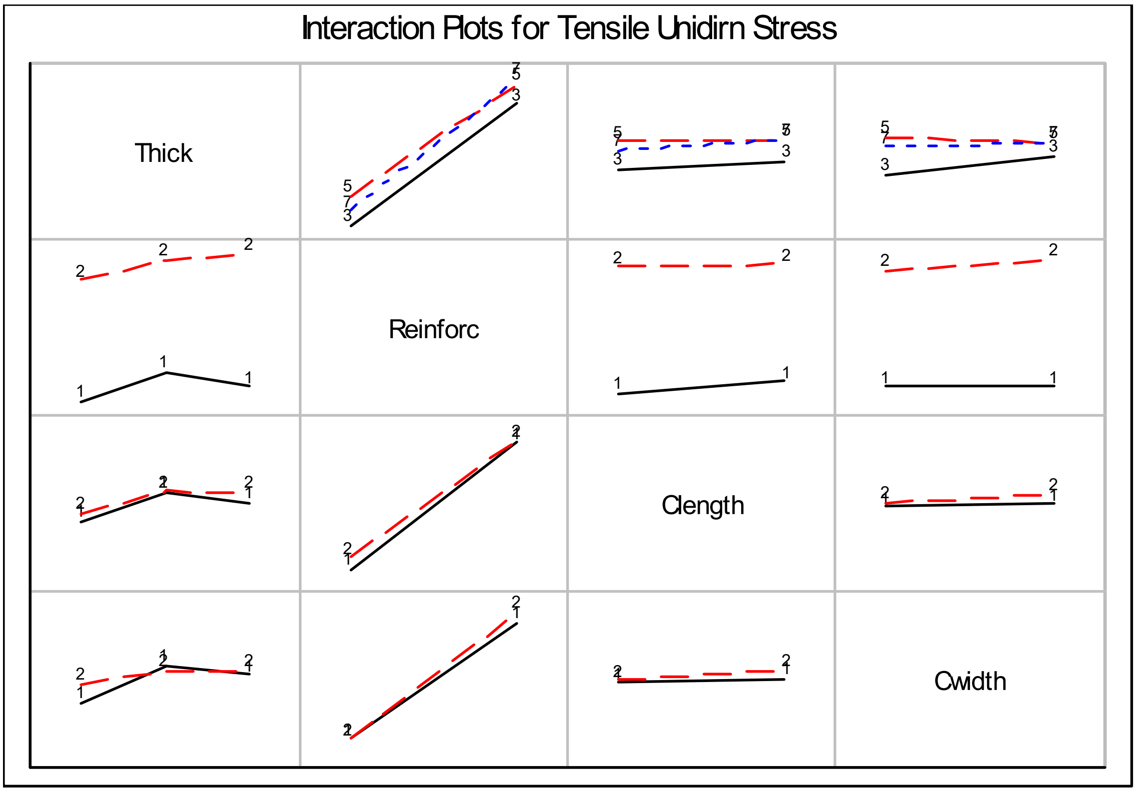

Fig. 8. Tensile stress interactions plots.

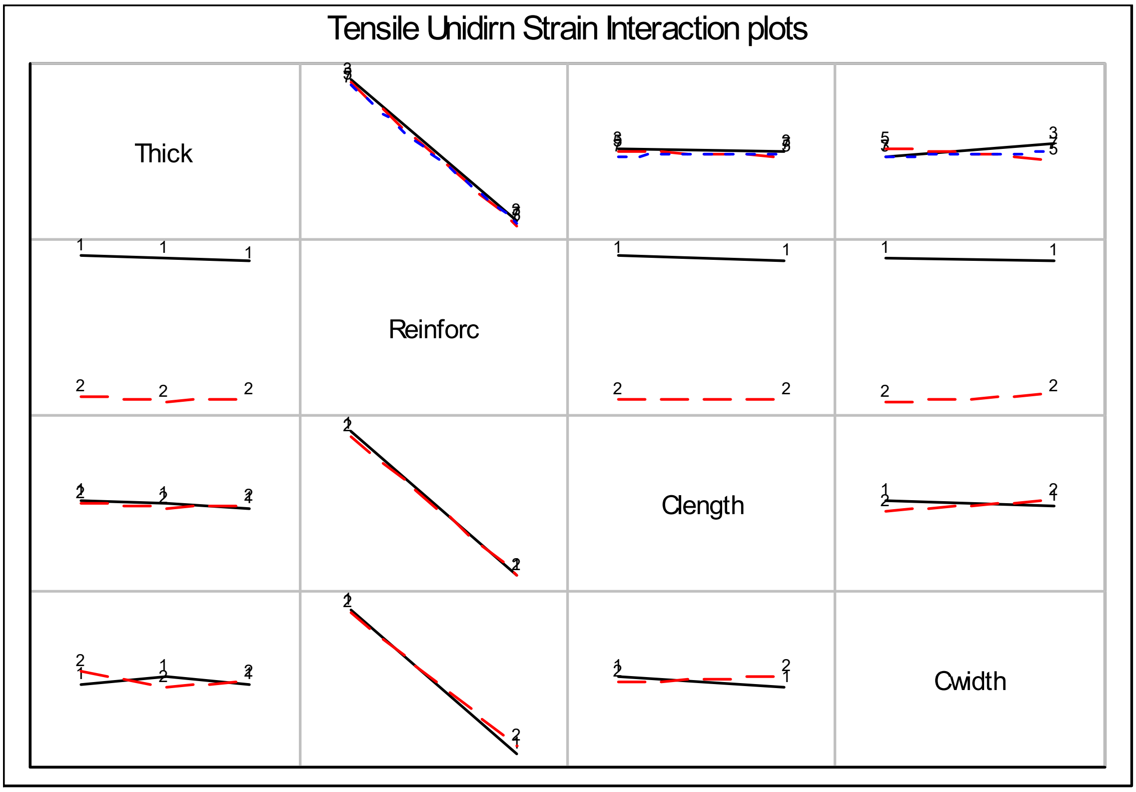

Fig. 9. Tensile strain interactions plots.

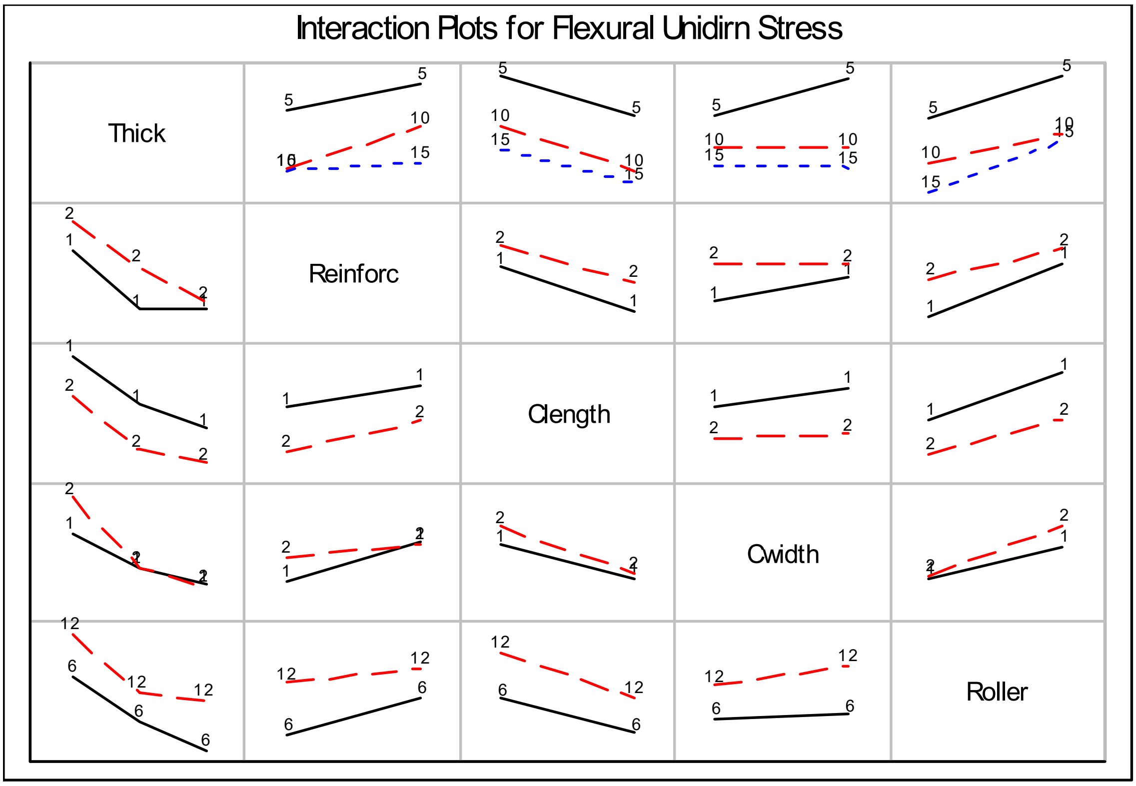

Fig. 10. Flexural stress interactions plots.

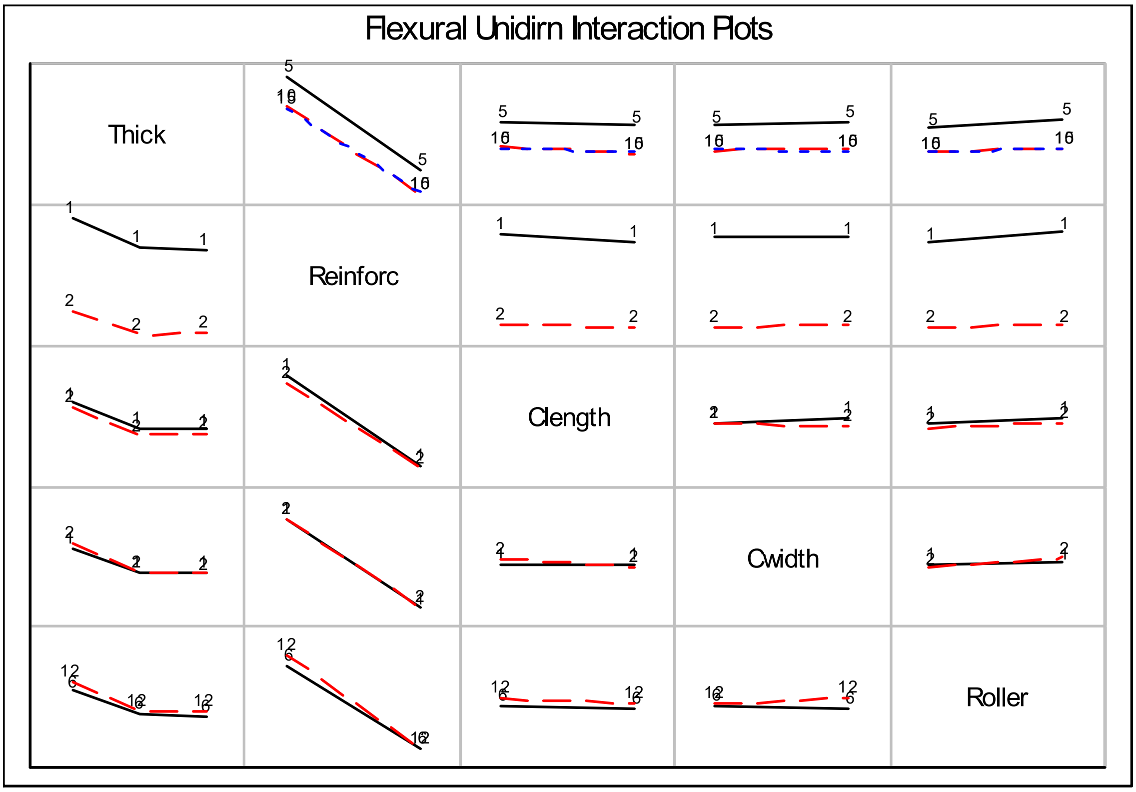

Fig. 11. Flexural strain interactions plots.

The *P*-values given in Table 10 help to identify important sub-plot factors. The important values in the table are underlined. The `*P*-value' given for each term in the proposed model is the probability that, in truth, the term makes no contribution to the response and the value observed is due solely to random error or scatter, i.e. `by chance'. Hence a very small *P*-value indicates that the size of the corresponding term is statistically important; in engineering applications a *P*-value $\leqslant 0.05$ is often taken as `statistically significant' and $\leqslant 0.01$ as `highly significant'. The trends may be seen in the main effects and interaction plots already included in the text.

	Tensile		**Flexural**	
Source	**Stress**	**Strain**	**Stress**	**Strain**
Clength	0.109	0.722	<u>0.000</u>	<u>0.000</u>
Cwidth	<u>0.007</u>	<u>0.048</u>	<u>0.023</u>	<u>0.021</u>
Roller	~	~	<u>0.000</u>	<u>0.000</u>
CLength*Cwidth	0.295	<u>0.030</u>	0.267	<u>0.000</u>
CLength*Roller	~	~	0.387	0.542
CWidth*Roller	~	~	0.175	<u>0.000</u>
Thick*Roller	~	~	0.360	<u>0.014</u>
Reinforc*Clength	0.155	0.600	0.825	0.119
Reinforc*Cwidth	0.094	0.132	0.064	0.283
Reinforc*Roller	~	~	0.136	<u>0.000</u>
R-Squared	98.00%	98.10%	89.20%	99.70%
Sub-Plot C.V.	0.0398	0.0570	0.0607	0.0302

Table 10. Sub-plot *P*-values for unidirectional tests

The value of `*R*-Squared' is the percentage of the variation in the response explained by the model.

5. Discussion

5.1. Statistical model

The models postulated fit the data well and this shows that the appropriate terms have been included. This is, in part, due to relatively high quality, laboratory methods used to produce specimens. The variation of other parameters that could affect the strength of the coupons, such as void content and misalignment of the fibres, for example, have been well controlled within each laminated panel. This is reflected in the coefficients of variation of around 5% in Table 10 which are relatively low for marine composites.

The coefficient for flexural stress is higher than that for flexural strain, and this suggests that the measurement of strain is more accurate than that of stress for these tests. Strain is measured directly by the strain gauges used, whereas stress is obtained indirectly through calculations involving other parameters. Additionally, the calculation of flexural stress involves an approximation of the effect of large central deflections of the specimen.

Despite the fact that the absence of any replication of laminates does not allow a thorough statistical examination of the whole-plot variation, a comparison of the whole-plot and sub-plot percentages of the total sums of squares in Table 8 shows that, in general, the whole-plot effects are larger than the sub-plot terms. The whole-plot terms refer to effects that occur due to variables that changes between panels, and the sub-plot terms refer to effects that occur due to variables that change within panels. Hence, these values suggest that not only are the effects of these two groups of variables different, but that those between panels are more important. The exception to this trend is seen in the analysis of the tensile failure strain measurements. Here the whole-plot sums of squares are an order of magnitude less than the sub-plot value, and this is thought to be due to the lack in precision in these comparisons as described above.

The sums of squares due to the reinforcement variable have been removed and hence the whole-plot values reflect the thickness terms in the model. Any differences between the properties of laminates produced with different thicknesses could be directly due to the effect of thickness, or could be a result of differences in the manufacture of the separate panels. It is not possible to distinguish between these two effects as the panels were not replicated; the two effects are said to be *confounded*.

5.2. Reinforcement, length, width and thickness

The effect of the reinforcement used is as expected from the published strength data of unidirectional E-glass/epoxy and high strength carbon/epoxy. The glass specimens failed at a lower stress than carbon (Fig. 4 and Fig. 6), but at a higher strain value (Fig. 5 and Fig. 7), for both tensile and flexural tests. However, the average compressive flexural failure stress of the carbon coupons is only marginally greater than that of the glass specimens.

The lack of a definite trend in the effect of thickness on the tensile strengths (Fig. 4 and Fig. 5), and the fact that the specimens failed near to the jaws, suggests that these effects are more likely due to production variability, than to statistical strength theories as described in the review part of this paper [1].

The flexural tests show a loss in strength with increasing thickness (Fig. 6 and Fig. 7). This could be due to statistical strength theory, a direct result of the size of the specimen, i.e. a size effect. However, this trend is not as simple as it would seem. There are different trends for carbon and glass reinforced specimens (Fig. 10), and although the flexural failure strain is less for the 5-ply than for the 10-ply specimens, it is approximately equal for the 10-ply and 15-ply specimens. Also, the failure of the specimens at the loading rollers does not fit in with the assumptions of this theory.

These effects might also be explained by manufacturing effects which are confounded with (i.e. indistinguishable from) thickness. This may be thought of as an effect of `scale', i.e. a lower quality specimen is obtained when a thicker laminate is produced. A plausible reason for this is that when there are more wet, and hence unstable, plies, the fibres of the top lamina (where the failure of the flexural specimens occurred) are more difficult to keep straight as they are `wetted-out' by hand.

There is a decrease in both failure stress and strain with increasing length for the flexural tests. This could be due to a `size effect' dependent on length such as that described in Ref. [1], and may be related to a similar effect of thickness mentioned above. Although the fact that the failure of the flexural specimens occurred under the load roller does not fit in with the assumptions of the Weibull strength theory (see Ref. [1]), and other interactions should be considered, it is a worthwhile to estimate the appropriate Weibull moduli here. Both of these effects were very similar for both carbon and E-glass composites and so the results from specimens reinforced with both materials may be `pooled' (treated as one group).

Similarly, the effect on the failure stress and strain of increasing the length of a specimen is very similar at each thickness, and hence the results may be averaged across thickness. For an increase in length by a factor of 1.39 there is a decrease in strength from 811 to 725 MPa. The corresponding drop in failure strain is from 1.73% to 1.64%. Assuming that the statistical strength theory is applicable here, this would give Weibull moduli based on length of approximately 3 for stress and 6 for strain. To compare these values with those in the literature for the volumes of geometrically scaled specimens it is appropriate to multiply them by a factor of three, giving Weibull moduli of 9 and 18 for failure stress and strain respectively. Despite the failure modes and the complicating interactions, these values are comparable (considering the different nature of the composites considered here) with those quoted in the literature.

For both test methods, the effects of width seen for failure stress and strain used are strongest in the smallest (i.e. thinnest) specimens, where an increase in width gives a higher failure strength (Figs. 8–11). For the tensile tests, a possible explanation for this is that the lower width of the 3-ply specimens (3.75 mm) is of the same order as the fibre bundle width of the unidirectional tapes used (2–3 and 4–5 mm for the carbon and glass reinforcements respectively), and hence a significant proportion of the bundles may be cut or damaged at the edges of the narrowest coupons, weakening them. However, the smallest flexural coupons were 12.5-mm wide, so it would imply that flexural testing is far more sensitive to fibre bundles edge damage, if the same explanation applies to these tests.

The interactions between the effects of length and width on the failure strains (Fig. 9 and Fig. 11) are not easily explained. Opposite trends are seen for the tensile and flexural tests, and the equivalent effects for both failure stresses are not significant.

5.3. Roller diameter

The flexural specimens tested with a larger support roller diameter gave higher strength values (Fig. 6 and Fig. 7). The statistical analysis of the data shows that this is a very strong effect for both failure stress and strain (Table 10). An initial statistical analyses of the strengths of the E-glass and carbon coupons considered separately failed to recognise this effect, it was only later when all the data was pooled that the importance of this effect became apparent. Similarly, an earlier study using 24 specimens of the same E-glass/epoxy material system [8] concluded that this factor did not affect the strength (although this deduction was not statistically based). This highlights the danger of using small numbers of coupons to investigate the behaviour of composite materials without due considerations of the statistical implications.

The effect on the flexural failure strain of an increase in roller diameter is greater for a wider specimen (Fig. 11). This interaction effect, although statistically very significant (Table 10), is not easily explained. The effect of roller diameter on the flexural failure strain is observed to be greatest for the 5-ply specimens (Fig. 11). It would be intuitive to assume that, as the thickness and hence loads increased, the stress concentrations at the rollers would become more important. This could be the case, but at the higher loads the 12-mm diameter rollers may not be large enough to reduce these concentrations as effectively as they do for the 5-ply coupons. Alternatively, the damage to the top lamina may be critical in a 5 ply specimen, but not so for a 10- or 15-ply coupon, because of the differences in the percentage of plies damaged.

The failure strain of the E-glass specimens is more strongly affected by the roller diameter than that of the carbon specimens (Fig. 11). The failure strain and impact resistance of carbon are much lower than those for E-glass [14]. Hence, it is plausible that this effect occurs because the 12-mm rollers do not reduce the stress concentrations in the carbon coupons sufficiently to affect the failure strain. In this case this trend is also seen in the failure stress, although it is not statistically highly significant.

6. Conclusions

As expected the use of carbon fibres gave lower strains to failure and a higher tensile strength. The compression flexural strengths of the two reinforcements were found to be comparable. An effect of thickness for tensile strengths showed no real trends, and the same effect for flexural strength was complicated by interactions. These facts, together with the failure modes seen, indicate that manufacturing variation, which is indistinguishable from the thickness effect in this experiment, is responsible for some or even all of the apparent effect of thickness. Longer flexural specimens were found to be weaker, but the failure modes seen raise doubts as to the suitability of statistical strength theory. Other variable effects and interactions were also found to be important, showing that the question of the effect of size effects for composites is not straightforward. Despite these doubts it was possible to calculate Weibull moduli similar to those found in the literature for the flexural tests.

Previous work in the field of composites size effects has either made no statistical inferences about any apparent size effects, or has made simple statistical tests assuming the experimental variance to be constant throughout the experiment. Here an extensive statistical analysis of the data showed (to a high probability) which trends could be assumed to be `real', and which could be attributed to background variation. The structure of the statistical analysis was split into two pertinent parts, one considering the variation between the properties of separate panels (the `whole-plot' variation), and one considering the variation between coupons cut from the same panel (the `sub-plot' variation).

The variation between panels is seen to be important, and is generally greater than that within a panel. Also, the effects of the variables that require different panels for different values are important. The implications of this are that the properties of these hand laid-up laminates are sensitive to variations at the fabrication stage, and hence that scale effects due to the scale of production may be important, as opposed to statistically based size effects.

The use of statistically designed factorial experiments has been shown to be very efficient and through in terms of the information gained, and also to take into account the experimental scatter seen for the composites considered. The use of such methods is strongly advocated for use in this and other fields of composite research.

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Appendix A. Test data

Tensile Tests: In the specimen code `G' signifies glass and `C' signifies carbon

Specimen Code	Length (mm)	Width (mm)	Depth (mm)	Modulus (GPa)	Fail Defln (mm)	Fail Stress (MPa)	Fail Strain (%)
UNIT/3G/1A	52.5	3.69	1.53	25.4	3.2	549	2.180
UNIT/3G/1B	52.5	3.69	1.53	25.4	3.2	549	2.180
UNIT/3G/2A	52.5	7.49	1.53	25.5	3.0	508	2.080
UNIT/3G/2B	52.5	7.49	1.52	24.9	2.9	492	1.980
UNIT/3G/3A	105	3.71	1.42	29.2	4.3	513	1.910
UNIT/3G/3B	105	3.71	1.39	27.5	3.5	528	1.900
UNIT/3G/4A	105	7.49	1.41	26.5	4.3	580	2.190
UNIT/3G/4B	105	7.50	1.40	26.7	3.9	543	2.050
UNIT/3C/1A	52.5	3.73	1.26	82.1	2.2	793	0.960
UNIT/3C/1B	52.5	3.71	1.26	85.1	2.3	824	0.960
UNIT/3C/2A	52.5	7.49	1.26	88.3	3.0	898	1.030
UNIT/3C/2B	52.5	7.46	1.25	84.0	2.6	901	1.080
UNIT/3C/3A	105	3.76	1.33	84.0	2.9	800	0.950
UNIT/3C/3B	105	3.75	1.33	92.2	3.4	842	0.930
UNIT/3C/4A	105	7.50	1.34	79.3	3.9	983	1.240
UNIT/3C/4B	105	7.52	1.34	85.2	3.4	908	1.080
UNIT/5G/1A	87.5	6.18	2.19	28.2	4.8	606	2.200
UNIT/5G/1B	87.5	6.25	2.14	29.5	4.5	598	2.075
UNIT/5G/2A	87.5	12.51	2.20	32.7	5.1	563	1.750
UNIT/5G/2B	87.5	12.36	2.23	29.5	5.6	617	2.160
UNIT/5G/3A	175	6.25	2.18	32.4	7.2	633	2.190
UNIT/5G/3B	175	6.21	2.19	34.6	6.8	651	1.925
UNIT/5G/4A	175	12.47	2.25	31.5	7.0	624	1.960
UNIT/5G/4B	175	12.49	2.21	31.6	7.2	629	2.050
UNIT/5C/1A	87.5	6.26	2.12	93.1	4.3	957	1.038
UNIT/5C/1B	87.5	6.27	2.12	91.4	4.2	948	1.038
UNIT/5C/2A	87.5	12.44	2.06	94.8	4.9	956	1.000
UNIT/5C/2B	87.5	12.42	2.10	97.8	5.3	896	1.000
UNIT/5C/3A	175	6.20	2.09	90.8	4.6	918	1.000
UNIT/5C/3B	175	6.18	2.08	95.3	4.6	918	0.968
UNIT/5C/4A	175	12.48	2.11	94.6	5.4	883	0.941
UNIT/5C/4B	175	12.49	2.13	100.4	5.2	926	0.928
UNIT/7G/1A	122.5	8.51	3.18	28.3	6.5	527	1.990
UNIT/7G/1B	122.5	8.59	3.10	28.5	6.7	548	2.000
UNIT/7G/2A	122.5	17.46	3.09	28.9	6.7	558	1.950
UNIT/7G/2B	122.5	17.41	3.12	29.5	7.1	585	2.050
UNIT/7G/3A	245	8.66	3.07	30.1	8.6	587	1.950
UNIT/7G/3B	245	8.70	3.09	29.3	8.6	595	2.075
UNIT/7G/4A	245	17.48	3.10	29.5	9.6	600	2.025
UNIT/7G/4B	245	17.44	3.12	29.2	9.6	597	2.050
UNIT/7C/1A	122.5	8.71	3.00	94.5	5.0	918	0.970
UNIT/7C/1B	122.5	8.59	2.98	94.3	5.2	938	1.000
UNIT/7C/2A	122.5	17.29	2.98	90.8	6.1	915	0.988
UNIT/7C/2B	122.5	17.48	3.02	91.7	6.4	945	1.025
UNIT/7C/3A	245	8.68	2.98	100.7	6.4	935	0.925
UNIT/7C/3B	245	8.72	2.97	93.4	6.7	956	1.025
UNIT/7C/4A	245	17.41	2.97	89.5	7.6	938	1.050
UNIT/7C/4B	245	17.45	2.99	91.8	7.6	963	1.050

Flexural Tests: In the specimen code `G' signifies glass and `C' signifies carbon

Specimen Code	Length (mm)	Width (mm)	Depth (mm)	Roller (mm)	Modulus (GPa)	Fail Defln (mm)	Fail Stress (MPa)	Fail (%)
UNIF/5G/1A	54	12.50	2.37	6	26.0	6.9	698	2.700
UNIF/5G/1B	54	12.44	2.34	6	27.7	7.2	756	2.750
UNIF/5G/2A	54	24.98	2.33	12	29.3	7.8	1034	3.150
UNIF/5G/2B	54	25.01	2.35	12	30.5	8.1	994	3.150
UNIF/5G/3A	75	12.48	2.3	12	27.0	14.2	756	2.900
UNIF/5G/3B	75	12.46	2.3	12	28.2	14.8	801	2.900
UNIF/5G/4A	75	24.68	2.16	6	35.0	15.8	813	2.600
UNIF/5G/4B	75	24.76	2.14	6	32.7	14.0	760	2.750
UNIF/5C/1A	54	12.49	2.21	12	82.0	3.1	928	1.125
UNIF/5C/1B	54	12.41	2.21	12	82.5	3.0	965	1.150
UNIF/5C/2A	54	24.55	2.2	6	77.7	3.6	928	1.175
UNIF/5C/2B	54	24.91	2.17	6	74.6	3.6	900	1.175
UNIF/5C/3A	75	12.50	2.17	6	71.2	5.9	784	1.088
UNIF/5C/3B	75	12.46	2.19	6	73.4	6.0	849	1.150
UNIF/5C/4A	75	25.05	2.31	12	70.1	6.2	865	1.200
UNIF/5C/4B	75	24.99	2.29	12	71.5	6.5	867	1.200
UNIF/10G/1A	108	25.00	4.46	6	31.8	13.6	736	2.400
UNIF/10G/1B	108	24.84	4.49	6	32.6	13.6	730	2.375
UNIF/10G/2A	108	49.83	4.46	12	33.9	14.0	800	2.500
UNIF/10G/2B	108	50.03	4.49	12	33.3	14.0	800	2.525
UNIF/10G/3A	150	24.65	4.03	12	32.5	27.0	676	2.200
UNIF/10G/3B	150	24.43	4.03	12	28.9	28.8	661	2.350
UNIF/10G/4A	150	49.85	4.06	6	32.2	25.6	588	2.150
UNIF/10G/4B	150	49.70	4.06	6	32.6	27.0	585	2.100
UNIF/10C/1A	108	25.07	3.62	12	117.4	4.6	857	0.700
UNIF/10C/1B	108	25.04	3.70	12	112.5	4.6	821	0.700
UNIF/10C/2A	108	50.05	3.44	6	111.2	6.2	836	0.750
UNIF/10C/2B	108	49.96	3.55	6	102.9	6.0	756	0.750
UNIF/10C/3A	150	24.64	3.45	6	101.9	10.2	821	0.750
UNIF/10C/3B	150	25.57	3.50	6	93.8	10.2	641	0.665
UNIF/10C/4A	150	49.84	3.73	12	101.7	9.8	791	0.750
UNIF/10C/4B	150	49.88	3.57	12	110.2	10.4	799	0.715
UNIF/15G/1A	162	37.51	6.67	12	33.7	21.8	782	2.450
UNIF/15G/1B	162	37.46	6.68	12	34.2	21.4	774	2.300
UNIF/15G/2A	162	74.88	6.69	6	32.7	21.0	669	2.150
UNIF/15G/2B	162	74.95	6.68	6	32.0	20.6	677	2.250
UNIF/15G/3A	225	37.43	6.59	6	30.5	38.4	595	2.250
UNIF/15G/3B	225	37.51	6.59	6	29.6	37.0	589	2.250
UNIF/15G/4A	225	75.01	6.67	12	32.2	38.4	683	2.300
UNIF/15G/4B	225	75.00	6.19	12	37.4	38.4	795	2.300
UNIF/15C/1A	162	37.53	6.07	6	91.0	8.5	702	0.750
UNIF/15C/1B	162	37.53	6.14	6	92.7	7.6	743	0.800
UNIF/15C/2A	162	74.48	6.13	12	92.6	8.8	798	0.850
UNIF/15C/2B	162	75.00	6.16	12	91.1	8.6	785	0.850
UNIF/15C/3A	225	37.52	5.88	12	97.4	14.0	729	0.725
UNIF/15C/3B	225	37.55	6.01	12	97.5	13.6	734	0.750
UNIF/15C/4A	225	75.04	5.98	6	96.4	7.7	650	0.665
UNIF/15C/4B	225	75.04	6.15	6	83.9	7.0	559	0.665