Size and scale effects in composites: I. Literature review

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Abstract

This paper is concerned with a review of the size and scale effects involved in the prediction of strength of fibre-reinforced-plastic (FRP) composite materials and structures. The review covers the basic principles in the establishment of scaling laws and the application of the Buckingham-Pi theorem. An analysis of various theories used in categorising size effects, such as the weak link, extended weak link and fracture mechanics approaches, is presented. This is followed by an examination of the literature devoted to scaling issues in FRP composites.

Keywords

Scale effects, Size effects, FRP, Composites, Dimensional analysis, Weibull analysis

1. Introduction

The relatively recent introduction of FRP composite technology, together with the large range of materials used and being introduced, mean that a broad design base, such as that available for many metals, has not yet been compiled for FRP materials. Hence much testing of composite components has to be carried out either on full-scale prototypes, or, in order to save both time and expense, on small-scale models by using the principles of dimensional analysis. It follows therefore, that any discrepancies encountered whilst scaling from model to full size (i.e. any size effects) should be both identified and understood. Similarly, much of the design of composite components is based on material properties derived from small laboratory scale coupons.

It has been thought for some time that a strength ‘size effect’ may exist for some composites, which is usually (but not exclusively) detrimental with increasing size. This is thought to be due to the increased probability of a larger specimen containing a flaw large enough to lead to failure. However, an accurate quantitative description of such effects, or even firm evidence of their existence, has proved elusive. These problems may be compounded by the fact that a separately manufactured specimen will not necessarily have the same properties as a comparable specimen cut from the full-scale structure. This latter effect is due more to the scale of production than to the actual size of the composite laminate considered. Hence it may be helpful to think of this as a ‘scale effect’ rather than a ‘size effect’.

The existing studies of this phenomenon have considered various different areas and approaches, all under the general heading of ‘Composites Size Effects’. The aims of this paper are two-fold; firstly to provide a thorough and structured summary of the terms and theories used in the literature, and then secondly to review this literature. This review will lead to the methodology used in, and provide the background to, an extensive study of ship-building composites which uses a novel approach described in two companion papers [1, 2].

2. Size effects

There are many texts concerning experimental modelling [3] the theory of models [4], and the field of dimensional analysis [5]. In order for the use of experimental scale model testing to be successful it is imperative that the unique relationship between the behaviour of the model and that of the prototype is well understood. It must be known that the model and prototype obey the same physical laws and that all the relevant features are correspondent if the model data are to be extrapolated to full scale. The unique relationship between model and prototype is broadly referred to as *similarity* and the conditions required to ensure similarity are developed using a technique known as dimensional analysis which is based on our concepts and conventions of measurements and observations.

2.1. Dimensional analysis

The use of dimensional analysis is directed towards finding pertinent non-dimensional combinations of variables for the physical system. These terms are subsequently employed in order to ascertain the required relationships between model and prototype.

The basis of the mechanics of dimensional analysis is the use of non-dimensional groups or *Pi terms* after the 'Pi-theorem' as set out by Buckingham [6]. The use of such groups enables similarity conditions between model and prototype to be established. The choice of appropriate Pi terms requires an understanding of both the physical processes and the relevant variables of the system. Omission of any variables involved in the system processes will result in errors in the prediction of prototype behaviour from model experiments. Conversely, selection of unnecessary or uninvolved variables may not affect the validity of the model, but may entail unnecessary complexity, possibly obscuring the physical relevance of the non-dimensional groups developed.

2.2. The theory of models

Once a set of Pi terms has been identified the next step is to use them to give the relationship between model and prototype behavior. It is convenient to consider two types of Pi terms, the test parameter and the design parameters or conditions. The test parameter is the Pi term containing the variable which is to be predicted and the design conditions are those containing the other system variables. As is normal engineering practice the system or prototype is expressed as a relationship between the parameter to be predicted and those which can be measured;

$$\Pi_{1p} = F\left(\Pi_{2p}, \Pi_{3p}, \Pi_{4p}, \ldots \Pi_{Sp},\right) \qquad (1)$$

where Π_{1p} is the test parameter and Π_{2p} to Π_{Sp} are the design conditions pertaining to the prototype.

Since Eq. (1) is entirely general it also applies to the model system since the latter is a function of the same variables;

$$\Pi_{1m} = F\left(\Pi_{2m}, \Pi_{3m}, \Pi_{4m}, \ldots \Pi_{Sm},\right) \qquad (2)$$

where the subscript m denotes model.

Therefore the relationship between prototype and model may be expressed as;

$$\frac{\Pi_{1p}}{\Pi_{1m}} = \frac{F(\Pi_{2p}, \Pi_{3p}, \Pi_{4p} \ldots \Pi_{Sp})}{F(\Pi_{2m}, \Pi_{3m}, \Pi_{4m} \ldots \Pi_{Sm})} \qquad (3)$$

Take the case where that all the test parameters are equivalent between prototype and model;

i.e.

$$\Pi_{ip} = \Pi_{im} \qquad (4)$$

Since the function *F* is the same for model and prototype, it follows that;

$$\Pi_{1p} = \Pi_{1m} \qquad (5)$$

In this case the model and prototype are said to be completely similar.

2.3. Distorted models and size effects

If one or more of the Pi terms differ between model and prototype scales then the model would be said to be *distorted* with respect to this Pi term. If this term is purely geometrical the model would be said to be geometrically distorted and this would be readily apparent from the fact that the model would be a different shape to the prototype. Geometric distortion is the most common in static structure modelling but others include loading and material properties distortion. In this case some recourse must be taken to reconstitute the relationship between model and prototype. Usually additional knowledge in the form of an equation not used in the dimensional analysis performs this task.

Once the analysis has been completed the prototype behaviour may be predicted from the experiments performed on the model. It is often the case that on construction of the prototype these predictions are found to be inaccurate to some degree. This is often referred as a 'scale effect', since the scale of the system appears to affect its behaviour. However this is simply a name given to an unknown phenomenon which has not been included in the initial appraisal of the system and hence the scaling relationship used. In other words the model is still distorted since we have not allowed for all the features of the system considered David and Nolle [3] suggest some reasons for such differences between model and prototype behaviour;

1. Some effect may be insignificant at prototype size but significant at model size or vice versa.
2. There may be a change in behaviour.
3. Measurement or construction accuracy may be different for different scales.
4. The material properties are affected by scale. This may be due to differences in fabrication methods, for example.

From a design point of view such scale effects may be allowed for using previous model and prototype scale experimentation experience to give engineering factors. A more scientific approach is to carry out experiments on a range of scaled systems with the view to use the results to formulate and verify a theoretical explanation for the model distortion.

3. Analysis of strength size effects

3.1. Weakest-link theory

Statistical strength theory or statistical weakest link theory has formed the basis of conventional brittle fracture study for many years. The concept of the weakest link was first used by Pierce [7] to investigate the strengths of long lengths of cotton yarns by considering them to made up of shorter lengths linked together. Shortly after this study, Tucker [8] applied the same concept to concrete. Weibull [9] made great advances in the subject, giving his name to the most widely used distribution used in weakest link theory and showing that the theory could be applied to many brittle materials.

Weakest link theory is based on the assumption that the material is made up of smaller elements linked together and that failure of the material as a whole occurs when any one of these elements or 'links' fail. The probability of failure of each link subjected to a stress increase from 0 to σ is described by the distribution function F(σ). The probability of survival of that link is then given by;

$$S(\sigma) = 1 - F(\sigma) \tag{6}$$

It is also assumed that $F(\sigma)$ describes the strength distribution for every element and that each $F(\sigma)$ is an independent randomly distributed variable. The probability of survival of n elements in series is then given by;

$$S_n(\sigma) = \left[1 - F(\sigma)\right]^n \tag{7}$$

Hence the probability of failure of a chain of n elements is given by:

$$F_n(\sigma) = 1 - \left[1 - F(\sigma)\right]^n \tag{8}$$

The function $F(\sigma)$ may be described generally as,

$$F(\sigma) = 1 - \exp\left[-\varphi(\sigma)\right] \tag{9}$$

Eq. (8) forms the basis of statistical weakest link theory. A specific form of $\phi(\sigma)$ was put forward by Weibull [9] and is still used widely today. This function has become known as the 'Weibull distribution' and is given by:

$$\varphi(\sigma) = \left(\frac{\sigma - \sigma_u}{\sigma_0}\right)^m \tag{10}$$

Where σ_u is the threshold stress below which failure does not occur and σ_0 and m are called the *scale parameter* and the *shape parameter*, respectively.

This form is termed the three parameter distribution. For the strength of a material subjected to an increasing load from zero to σ there is zero chance of failure only when there is no applied load. Hence, σ_u is usually taken to be zero and the two parameter form is used. This leads to a probability of failure of n elements in series of;

$$F_n(\sigma) = 1 - \exp\left[-n\left(\frac{\sigma}{\sigma_0} \right)^m \right] \tag{11}$$

Considering a volume of material comprising of small elemental volumes, δV, instead of a chain of 'links' gives;

$$F_V(\sigma) = 1 - \exp\left[-\int \left(\frac{\sigma}{\sigma_0} \right)^m dV \right] \tag{12}$$

For tensile testing the stress σ is uniformly distributed through the material volume. Initially assuming this simple case gives;

$$F_V(\sigma) = 1 - \exp\left[-V\left(\frac{\sigma}{\sigma_0} \right)^m \right] \tag{13}$$

In order to use this equation to describe experimental data it is convenient to express Eq. (13) in a linear form. Rearranging and taking logarithms twice gives;

$$\ln\left[\ln\left(\frac{1}{1 - F_V(\sigma)} \right) \right] = m\ln(\sigma) - m\ln(\sigma_0) + \ln(V) \tag{14}$$

Hence a plot of $\ln(\sigma)$ versus the left hand side of this equation for N replications of an experimental strength test for a given volume of material, V, will give a linear relationship if the material strength variability is described by the Weibull distribution. From the slope and y-axis intercept of this line both the shape and scale parameter respectively may be estimated. Varying the volume of material translates the line vertically.

The calculation of $F_V(\sigma)$ for a set of experimentally obtained stresses, σ, can be carried out using a statistical approximation technique as described by Weibull [9]. Here the N values of σ obtained for a given volume, V, are arranged in ascending order, and for the i^{th} value;

$$F_V(\sigma) = \frac{i}{N + 1} \tag{15}$$

Alternatively, the shape parameter m may be approximated from the coefficient of variation (C.V.) of the data set, as stated by Hitchon and Phillips [10]:

$$m \approx \frac{1.2}{C.V.} \tag{16}$$

Considering the more general case of a varying stress field through the material volume, Eq. (12) becomes:

$$F_V(\sigma) = 1 - \exp\left[-\int \left(\frac{\sigma(x,y,z)}{\sigma_0} \right)^m dV \right] \tag{17}$$

On integration we can express this generally as:

$$F_V(\sigma) = 1 - \exp\left[-VK_s\left(\frac{\sigma_r}{\sigma_0}\right)^m\right] \tag{18}$$

where K_s is a factor dependent upon the stress distribution and σ_r is a reference stress at a specific point in the material. For tensile tests the stress distribution is uniform and hence K_s has a value of one. The derivation of K_s for four-point bending with a load span of one third of the support span results in [11];

$$K_s = \frac{m+2}{(m+1)^2} \tag{19}$$

If the strength distribution of a material is described by Weibull theory then it is possible to correlate the strengths of specimens or components of differing size. An assumption is made that the values of the shape and scale parameters m and σ_0 are material constants, independent of the size of the specimen and its stress field. Considering Eq. (13) for the same probability of failure for two specimens with identical stress distributions gives;

$$\frac{\sigma_2}{\sigma_1} = \left(\frac{V_1}{V_2}\right)^{\frac{1}{m}} \tag{20}$$

This equation directly links strength to volume and hence quantifies the size effect. A logarithmic plot of stress versus volume gives a straight line relationship of slope −1/m, as shown in Fig. 1.

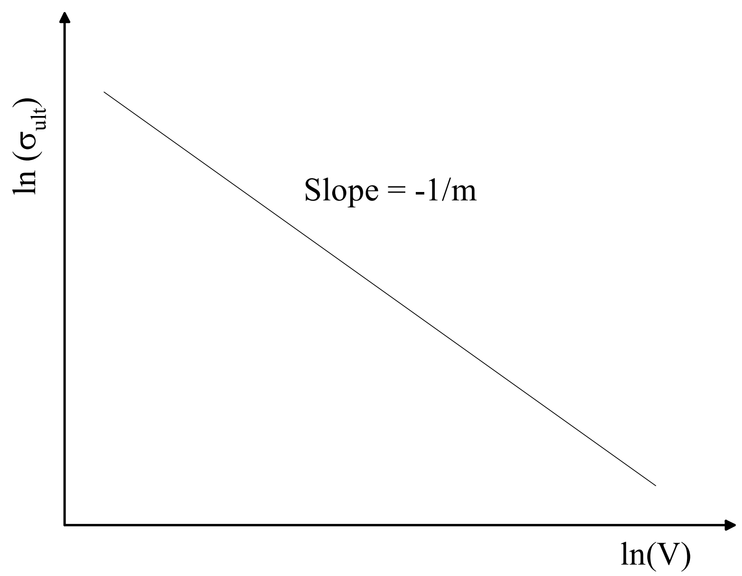

Fig. 1. Logarithmic plot of a strength size effect.

We can extend this approach to include the effect of differing stress distributions by using Eq. (18) to give:

$$\frac{\sigma_2}{\sigma_1} = \left(\frac{K_{s1}V_1}{K_{s2}V_2}\right)^{\frac{1}{m}} \tag{21}$$

This equation again quantifies the size effect but also includes the influence of differing stress distributions.

For anisotropic materials such as fibre composites both the strength distributions and the effects of flaws in differing directions may not be the same. Integrating Eq. (12) over length instead of volume, for example gives;

$$\frac{\sigma_2}{\sigma_1} = \left(\frac{l_1}{l_2}\right)^{\frac{1}{m_l}} \tag{22}$$

Similarly for breadth and depth:

$$\frac{\sigma_2}{\sigma_1} = \left(\frac{b_1}{b_2}\right)^{\frac{1}{m_b}} \tag{23}$$

$$\frac{\sigma_2}{\sigma_1} = \left(\frac{d_1}{d_2}\right)^{\frac{1}{m_d}} \tag{24}$$

where m_1, m_b and m_d are not necessarily equal.

The theory as applied to anisotropic materials is referred to as *modified* weakest link theory.

3.2. Extensions of weakest-link theory for composite materials

Weakest-link theory accurately describes the failure of brittle materials. However, although most composite materials fail at very low tensile strains, final failure generally occurs after some damage accumulation. There is much available literature suggesting the constituent fibres of many composites do behave as brittle materials [12, 13]. This is one of the assumptions of the theory first put forward Rosen [14] and Zweben and Rosen [15], but damage accumulation is also taken into account. Further work by Harlow and Phoenix [16] and Smith [17] gave comprehensive exact mathematical solutions. However, these more complicated models also require the estimation of often difficult to measure parameters. Also, other sources of variation are likely to be comparatively large and so the simpler and more easily interpreted theory of Zweben and Rosen is outlined here, as described in more detail by Batdorf [18]. The bundle of fibres model, as first suggested by Daniels [19], is in fact an extension of simple Weibull theory. A chain of elements is again considered, but in this case the elements are assumed to consist of many fibres.

Daniels assumed that for a loose bundle of fibres when an individual fibre failed, the bundle as a whole did not fail due to redistribution of the load equally among the other fibres. In a similar manner to Daniels [19], Zweben and Rosen [15] hypothesised that when the first of these fibres failed the composite as a whole did not fail, because of load transfer by the matrix. However, they supposed that the load previously taken by the broken fibre is now transferred via the matrix only to the *adjacent* fibres, around the break and then back to the original fibre. A consequence of this shear transfer is that, for a certain length either side of the break, the failed fibre carries less load whilst those adjacent carry more. This is shown in Fig. 2.

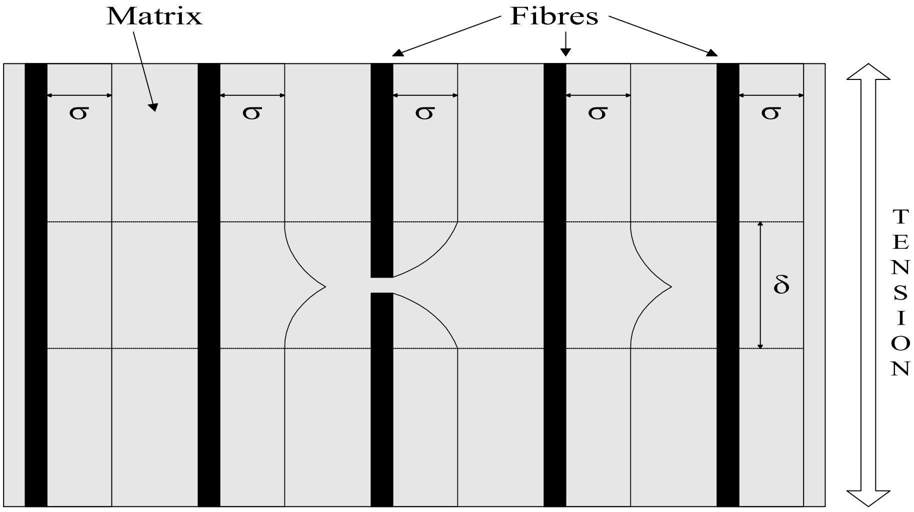

Fig. 2. Illustration of fibre load sharing.

The length over which this shear transfer occurs is known as the 'ineffective length' denoted by δ. Despite the fact that the adjacent fibres now carry more load, since δ is small the probability of a critical flaw occurring in this length is also small and hence failure is unlikely. Also the excess load is shared amongst all neighbouring fibres and so the load increase is not large.

This type of isolated fibre breakage is termed a 'singlet' and as the load is increased more of these will appear. As the load is further increased it becomes more likely that the over stressed parts of the adjacent fibres should themselves fail. When this occurs there are two adjacent broken fibres and this is termed a 'doublet'. Still further loading will give rise to more singlets and doublets and then 'triplets'. This continues until a critical 'multiplet' occurs and the process becomes unstable, resulting in the failure of the composite as a whole.

Considering the two parameter Weibull distribution:

$$\varphi(\sigma) = \left(\frac{\sigma}{\sigma_0}\right)^m \tag{25}$$

For the fracture of a fibre this may be interpreted as the number of defects unable to sustain a stress σ per unit length of fibre. Hence the number of singlets formed in N fibres of length L is:

$$Q_1(\sigma) = NL\left(\frac{\sigma}{\sigma_0}\right)^m \tag{26}$$

This assumes that the ineffective length δ is much less than the fibre length L. A further assumption is that N is large so that the number of flaws at the edges of the composite and hence not surrounded by other fibres is small.

In order to simplify the analysis the ineffective length for a singlet (1 is replaced by a conceptual 'effective length' λ_1. Each fibre adjacent to a singlet has a maximum increase in stress in the plane of the break. The effective length is that which, when subjected to this maximum stress, has the same probability of failure as the ineffective length subjected to the actual varying stress.

The total length of overloaded fibres surrounding the Q_1 singlets is hence $Q_1 n_1 \lambda_1$ where n_1 is the number of fibres around each singlet. Defining the ratio of σ_{Max} to σ as C_1 gives the number of failures expected in this length as:

$$Q_2 = Q_1 n_1 \lambda_1 \left(\frac{C_1 \sigma}{\sigma_0} \right)^m \tag{27}$$

This is the number of singlets converted to doublets at stress σ and may be generalised to higher order multiplets:

$$Q_{i+1} = Q_i n_i \lambda_i \left(\frac{C_i \sigma}{\sigma_0} \right)^m \tag{28}$$

This is not generally equal to the number of i-plets present at load σ since some will have been converted to higher order multiplets, the actual number is hence:

$$q_i = Q_i - Q_{i+1} \tag{29}$$

It can be seen from Eq. (28) that a logarithmic plot of Q_i against σ for the i^{th} multiplet will yield a linear graph of slope m_i. This is illustrated for i=1 to 4 in Fig. 3.

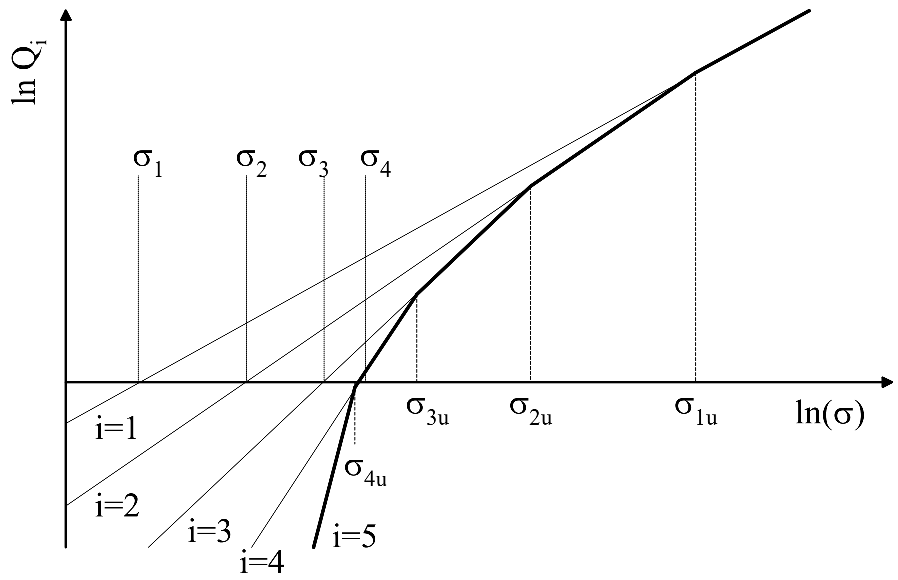

Fig. 3. Logarithmic plot of the bundle of fibres model.

From Eqs. (26) to (28) it is apparent that the number of multiplets is proportional to *NL*, which in a uniform unidirectional composite, is proportional to the composite volume, *V*. Hence a change in *V* translates the lines in Fig. 3 vertically, changing the values of the intercepts with the ln(σ) axis. This relationship may be represented on a logarithmic plot of failure stress against *NL*, as shown in Fig. 4. This plot is analogous to that for simple Weibull theory (see Fig. 1).

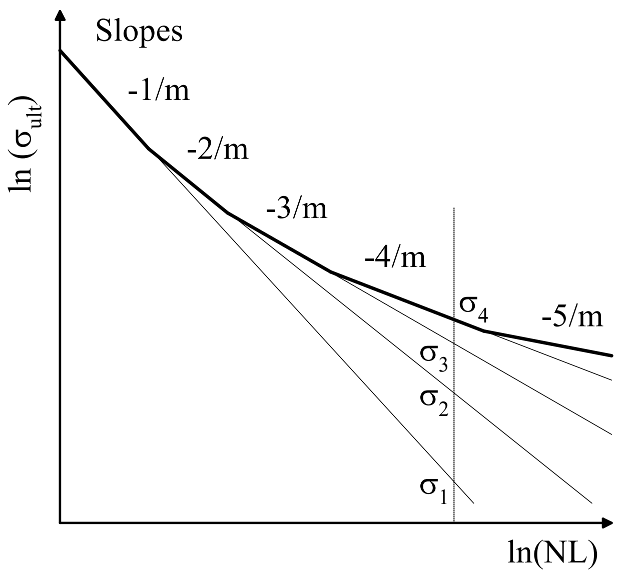

Fig. 4. Logarithmic plot of bundle of fibres model.

The failure line is again the bold line, the dashed line indicating the situation shown in Fig. 3 where failure occurs when the first quadruplet is formed. As the material volume is increased the failure stress again decreases, i.e. a strength size effect is present. The order of the critical multiplet is seen to increase with composite volume. Also, the dependence of strength on volume decreases as larger amounts of material are considered.

The example above is only an illustration, the exact form of the curve will change with the values of the parameters m, σ_0, n_i, λ_l and C_i. The predictions made using the model are thus highly dependent upon the estimation of these parameters.

3.3. Linear elastic fracture mechanics

For the simplest case of a crack of length 2a in an infinite plate subjected to a uniform stress σ, a linear elastic fracture mechanics (LEFM) approach [20] leads to the definition of the stress intensity factor, *K*. Failure of the plate occurs when this reaches a critical value, K_c giving a corresponding stress value σ_c:

$$K_c = \sigma_c \sqrt{\pi a} \tag{30}$$

This critical stress intensity factor is normally assumed to be a material property and is found experimentally by methods such as the tensile testing to destruction of a sample with a crack of known length.

Since it is assumed that the crack propagates when the stress intensity factor reaches its critical value then it would appear to be pertinent to include it in the dimensional analysis of the system, and this leads to:

$$\lambda_\sigma = \lambda_l^{-1/2} \tag{31}$$

This equation states that failure occurs at the same value of the critical stress intensity factor for both model and prototype then the failure stress will scale as one over the square root of length if there is geometric similarity. Thus strength decreases with size and a size effect has been described.

The system equation approach using Eq. (30) gives:

$$\frac{\lambda_\sigma \lambda_a^{1/2}}{\lambda_K} = 1 \qquad (32)$$

From this it is evident that if *K* is to remain constant and if Eq. (31) applies then the crack size, a, should scale as length. This would be expected from the condition of geometric similarity. However, this is a false assumption for most cases. For an advanced composite material, the critical crack or flaw is generally within the fibres. As is usually the case, if the same reinforcement is used for both model and prototype then the crack size is the same at both scales. This would also be true if the critical flaws were within the matrix phase, in the form of microcracks, for example. This point is further discussed in Section 4.4.

4. Strength size effects literature review

In his article entitled "Is There a Size Effect in Composites?" Zweben [21] stated that the question of the existence of a size effect in composites has been around since the 1960s. The fact that much work is still currently underway in the subject shows that conclusive evidence has not yet arisen to answer the question.

4.1. Brittle materials size effects

The weakest link theory has been used in the design of ceramics for some time and examples of the literature on this subject are numerous. Davis [22] gave an easily interpreted overview of the theories used and the mechanics of how to apply them.

The strength size effect of a natural anisotropic fibrous composite, wood, is analogous to that for man-made fibre reinforced plastics. This problem has also been approached using Weibull weakest link theory as early as 1966 by Bohannan [23]. Simple Weibull theory was applied to the decrease in strength with volume by Madsen and Nielson [24]. This theory was extended to allow for the anisotropic nature of wood using a modified Weibull theory in which the effects of length, width and thickness are considered separately [23, 25, 26].

4.2. Brittle fibres size effects

The consensus of work concerning individual filaments and bundles of fibres [12, 21, 27, 28] does indicate that there is a decrease in strength as length is increased, and also as the number of filaments increases. However, the description of these effects by the statistical theories used is by no means conclusive. For example, Moreton [12] describes only as 'fair' the fit of the weakest link model he used to describe the strength of carbon fibres.

Further extensions of these theories appear to have increased not only the complexity of the statistics involved but also of the interpretation of the results obtained. Watson and Smith [29] used the carbon fibre data of Bader and Priest [28] to fit more complicated models, but still admitted that the theory is not apparently satisfied for bundles. Despite their mathematically elaborate model, phrases such as '...but this is just a guess and major source of uncertainty' describing the estimation of the relevant parameters raises doubts as to the need for such detailed models. Padgett et al. [13] used a further development of Weibull theory only to obtain 'reasonable' fits to the same data. The mathematical simulations of Karbhari and Wilkins [30] used experimental data to give estimates of certain parameters but there is no experimental verification of their predictions. This pattern of more and more complicated statistical analyses with either very little experimental back-up or inconclusive correlation is also seen in the work concerning aramid fibres [31, 32].

4.3. Statistical strength theories

Various statistical aspects of the fracture of composites were discussed by Kelly and MacMillan [33]. Batdorf [18] provided an excellent overview of both simple weakest link and bundles of fibres models for fibrous composites. He took the view that, although the simplifications made by Zweben and Rosen [15] lead to a non-exact mathematical solution, the increase in accuracy achieved by such an exact solution is small when compared to the errors inherent in estimating the model parameters such as the ineffective length. He also recognised the advantages given to engineers of a model easily reconcilable with physical quantities over abstract statistical models. Exact mathematical results for the chain of bundles model were first obtained by Harlow and Phoenix [16, 34] and later, Smith [17] developed asymptotic approximations.

Not all the literature concerning the statistical fracture theories as applied to aligned composites attempts to verify the models derived and, as for fibres, conclusive evidence either for or against the model is not gained from the experimental data. The models appear to give adequate qualitative predictions but are quantitatively inaccurate. Model simplifications (such as fibre arrangement, load sharing and diameter assumptions) as well as the difficulties in estimating parameters (for example ineffective length and stress concentration factor) have been suggested as possible reasons for this. Another supposition is that the theory does not allow for the fact that sources of variation, other than those due to flaws, are inevitably present (see Section 3.3). Rosen [14] verified qualitatively the progressive and random nature of fibre fractures before final failure of a single layer glass laminate by experimental observation. Zweben and Rosen [15] analysed Rosen's strength data and found that their theory predictions correlated well with the data for small specimens. However, they questioned whether these results could be extrapolated to larger volumes such as those found in structures. Bartdorf and Ghaffarian [35] re-analysed the data of Bullock [36] and found that their model fitted only when the estimate of the ineffective length parameter was unrealistically large. Bader and Priest [28] were unable to reach any firm conclusions about the agreement with theory for data from single carbon fibres and impregnated bundles. In order to simplify the problem, Beyerlein and Phoenix [37] considered carbon fibres and simple micro-composites of four fibres in an epoxy matrix. However this approach, together with the inclusion of some measure of the variations in fibre diameter, still gave only a partial fit to the experimental data. Good predictions between the strengths of carbon fibres, minitows and tows were achieved by Batdorf [18] but the variation of strengths was much greater than expected. He suggested that this is due to sources of variation other than those due to flaws.

4.4. Carbon composites size effects

Most of the studies of composite size effects has been carried out in the aerospace field using pre-preg carbon/epoxy laminates and mainly use simple Weibull theory to explain any size effects seen.

A frequently quoted paper is that by Bullock [36] which described a study of two graphite/epoxy systems. Simple Weibull analysis was used to compare the strengths of single strand tows, tensile coupons and flexural three point bending specimens. The effect of volume between the similarly stressed tows and coupons was predicted very accurately by a Weibull theory that uses both volumes and stresses based on the fibres alone. Similarly, the theory predicts very well the differences in strength observed between the tensile and flexural coupons of equal volume by considering the different stress distributions present. Similar values for the Weibull shape parameter were obtained for tows, tensile coupons and flexural specimens (for 36, 27 and 13 specimens, respectively) for one of the material systems, and an average value of 24 was taken. However, for the other system, more variability in the strengths was observed and a value of 18 was taken for the shape parameter. Bullock suggested that sufficient specimens must be tested in order to estimate adequately the specific value of the shape parameter for the material considered.

The effect of fabrication route on the strength of CFRP was investigated by Hitchon and Phillips [10] who considered two types of both fibre and matrix. The specimens produced using pultrusion, hand lay-up, filament winding and pre-preg techniques were most conveniently tested using different test methods. Tensile, hoop-burst and three point flexural tests were carried out. Here Weibull theory only partially explained strength changes; tensile and hoop-burst results were well explained but flexural and tensile strengths were not reconciled. Variations in the shape parameter m (between 10.3 and 38.4) were cited as a possible reason for this; the Weibull analysis being carried out using an average value of 20. Changes in material properties between fabrication routes and failure mode differences between test methods were thought to be responsible for the observed variation in the shape parameter. Also the small number of specimens tested (between four and eight) was noted with reference to the confidence in the parameter estimates obtained. This statistical aspect was further investigated in the second part of the study which sought to resolve these problems by comparing the strengths of different sizes of filament wound hoop-burst specimens. No significant strength decrease ($p<0.01$) was seen, but it was noted that, for the small number of observations made, the size effects expected for the strength variability seen would be too small to discern. Hitchon and Phillips [10] postulated that Weibull theory may be more applicable when volume is changed through stressed fibre length rather than composite cross-sectional area, and they concluded that further investigation was required.

Tensile and flexural beam column tests of carbon/epoxy were the subject of a series of studies by Kellas and Morton [38], Jackson et al. [39] and Jackson and Kellas [40]. Various lay-ups were tested, initially using ply-level scaling, and a size effect noted for flexural and tensile testing which depended on the lay-up. No effect of size is seen on the initial stiffness. Both Weibull and LEFM theories were applied, the former giving better, but variable, correlation between theory and experiment. No attempt was made to estimate the Weibull shape parameter from the variation of the data, instead this was estimated using Eq. (33) and the strengths of two specimen sizes:

$$\frac{\sigma_2^{Ult}}{\sigma_1^{Ult}} = \left(\frac{V_1}{V_2}\right)^{1/m} \quad \textbf{(33)}$$

where σ^{Ult} is the specimen strength, V is the specimen volume, m is the shape parameter and subscripts 1 and 2 indicate the two different sizes of specimen considered.

Large variations of m (7.22 to 156 for the tensile tests, and 8.5 to 18.3 for the flexural) were observed across the different lay-ups. The fracture mechanics model was thought to be inappropriate due to the complex damage modes exhibited by the composite laminates. Failure mode transitions were noted as the size increased for both tensile and flexural coupons, and again this was dependent on lay-up. Sub-ply level scaling, whereby the number of fibres in each laminate is varied, was then applied to the same flexural beam column arrangement. This was found not to alleviate the strength scale effect and, in fact, the effect was amplified compared with the earlier ply-level scaled specimens.

Wisnom [41] observed a size effect for unidirectional carbon/epoxy for four-point bending and pinned-end buckling tests. A change in failure mode from tensile to compressive was seen with increasing size. Wisnom postulated that a greater size effect in compression than in tension caused the larger specimens to have lower compressive strengths than tensile strengths. Importantly, it was noted that the cure used heated plates for thin specimens, whereas an autoclave was used for the thicker specimens. Although visual inspection showed no difference in material quality, the different manufacture processes could provide a possible explanation for the size effects seen. A Weibull shape parameter of 25 was estimated from Wisnom's data using Eq. (33), but less scatter in the results than this suggests was seen. In order to explain this behaviour, a model of the composite between the extremes of a brittle solid and a loose bundle of fibres was postulated. This model predicted a size effect more dependent upon length than upon the other dimensions. Hence a second study of specimens of the same cross section, but with varying lengths using three-point bending tests was completed Wisnom [42]. Here a Weibull model with both volume and length terms was fitted to the data and found to account for the lower than expected variation. Wisnom draws attention to the caution required when comparing relatively small differences in strength based on observations from a small number of specimen tests. In further papers Wisnom [43, 44] reports a size effect for interlaminar tensile and shear strength measured using curved beam four point bending and short beam shear tests, respectively.

Grothause et al. [45] compared three and four point bending of carbon-fibre-reinforced plastic using Weibull theory. The stress concentrations at the loading rollers were found to influence failure mode, with steel rollers producing compressive failure and plastic rollers giving tensile failure. By considering tensile and compressive failures separately, Weibull theory was used to explain strength differences.

An investigation into scale effects for fatigue by Chou and Croman [46] also concerns graphite/epoxy. Here in-line holes drilled in the specimens were used to represent the 'links' in weakest link theory. Application of Weibull theory then allowed reasonable predictions to be made. Grimes [47] reviewed selected literature on the static and fatigue scale effects of graphite epoxy bonded and bolted joints.

Some work has been carried out on the scaling of the impact of carbon reinforced plastics [48–52]. The scaling of CFRP notched strength has also been studied by Shahib et al. [53].

Publications which consider the effects of scaling at a microstructural level include a study of defects by Wang [54] and studies into the relationships between ply thickness and damage accumulation by Crossman and Wang [55], Crossman et al. [56] and Lagace et al. [57].

4.5. Glass composites size effects

There is less information available concerning glass reinforced plastic composites, which are of far greater interest to the marine engineer. Camponeschi [58, 59] evaluated the effect of size on compression strength of carbon and glass composites for large naval structures. He postulated that although strength was observed to decrease with thickness, this could be attributed to fixture restraint effects. A size effect for the strength of glass fibres was found by Kies [60] but he stated that, with good design, a correspondingly large size effect is not seen in the strength of filament wound pressure vessels. He also states that, “The rather large discrepancy between virgin filament strength and strength in structure should not be regarded as due to fiber degradation but rather associated with unequal tensioning limitations due to resin, surface finishes, and design factors not yet optimised.” Elliot and Sumpter [61] considered a material common in the marine industry, woven roving glass/polyester. In this case no change in compression strength with size was found. However, a change was found for tensile tests, and again this was attributed to fixture effects. Interestingly, compressive strength was seen to vary through the thickness of the parent laminate from which the specimens were cut. The behaviour of woven roving glass/polyester was characterised in tension, compression, shear and flexure by Zhou and Davies [62, 63]. LEFM and simple Weibull theory were used to explain strength variations with size. The latter method was found to give better predictions of experimental results, although the lack of statistical analysis of small differences in strength obtained from small numbers of observations does not instil confidence in the conclusions proffered.

Crowther and Starkey [64] found a size effect in the fatigue of unidirectional glass reinforced epoxy and used Weibull statistics to explain this. They recognised that, “The success of this will depend on how sensitive fatigue life is to the difference in the manufacturing routes used to make small specimens and large components.”

4.6. Effects of ‘scale’

One reason for the concentration of the literature on size effects due purely to the amount of material could be that most of the work has been carried out in the aerospace field using pre-preg carbon/epoxy laminates. Here the material is fairly consistent and perhaps the effects of manufacturing are thought to be unimportant.

The significance of any size effects due to the scale of production rather than solely due to the size of the artefact are mentioned only briefly in very little of the literature on composites size effects. This issue is most comprehensively discussed by Zweben [21, 65] and Batdorf [18]. Most of the literature on the subject appears to be more interested in obtaining rigorous mathematical and empirical solutions to the theories. Other type of possible explanations for size effects, principally those arising from fabrication and production considerations, are mentioned only rarely. Zweben [21, 65] recognised them as important but then ignores them. Hitchon and Phillips [10] suggested that the reason for differences between their data and that of Bullock [36] is due to differences in the materials used and also note quality differences in batches of their own material, despite efforts to ensure that the manufacturing processes were identical. Crowther and Starkey [64] suggested that manufacturing differences may be important and that it would be useful to investigate these effects. A rare example of a study of the real life problem of scaling coupon data to full scale testing is the rather qualitative and specific work of Lowe and Satterly [66] concerning filament-wound glass/polyester spars for wind turbine blades. A more quality-control orientated paper by Karbhari et al. [67] suggested that the effect of the volume of production and material processing should be considered as well as geometric scale, but no specific details are mentioned. Scaling effects in nature were discussed by Wilkins [68] that might be relevant to composites, but again no direct comparisons are made.

5. Discussion

The statistical fracture theories used are well developed mathematically and are usually based upon the weakest link theory also known as 'Weibull theory'. This theory assumes brittle behaviour and has been found to satisfactorily describe the size effects seen for the strength of both ceramics and single constituent fibres of composite materials (such as those of carbon and glass). For the failure of bundles of fibres the simple fracture theories have been developed to give reasonable descriptions of the progressive failure mechanisms encountered.

However, when fibre reinforced plastic composites are considered the agreement between experiment and theory is by no means clear, and the literature is often contradictory. The theories are generally derived assuming simple tensile failure of unidirectional composites, but have been applied where more complex failure modes of laminates occurs. In some cases the theories have only been presented as a mathematical exercise without any reference to experimental data. The complexity of some of the models becomes redundant when their parameters become difficult or impossible to estimate, and gross approximations are required. Also, the advantages of further refining the mathematical model must be balanced against the assumptions made in the derivation of the theory (especially concerning the uniformity of the microstructure).

The LEFM model, although appearing in a number of publications, does not appear to describe the experimental data to which it is applied at all well. One reason for this could be that the dimensional analysis which produces this theory requires that any cracks in the composite are geometrically scaled with the specimen or component. Since the strength of the fibre reinforced plastic composites concerned is mainly fibre controlled, this is not plausible. There is no reason why the flaws in a larger laminate should be any different from those in a smaller one, if the same reinforcement is used in both. The theory also does not allow for the complex failure modes often seen in FRPs.

The studies in the literature, as a group, do not follow any particular pattern. The testing of composites is a complex subject on its own even before the issue of size effects is raised. Often the phrase, "composites size effects" is used to cover the entire spectrum of material properties, test methods, test parameters and material systems considered in all of the literature. For example, one study may consider the tensile testing of hand laid-up, 0°/90° carbon/epoxy laminates [38], whereas another may concern four-point flexural testing of pre-preg unidirectional carbon/epoxy [41]. Moreover, comparisons have been made within individual studies between composites manufactured using completely different processes such as filament winding, pultrusion, hand lay-up and pre-preg [10]. Considering these points, it is perhaps not unexpected that the results are often contradictory. It is neither effective nor efficient to approach a problem involving a large number of variables by considering each in turn and in isolation.

Despite the lack of a general consensus of opinion on the nature of strength size effects for fibre reinforced plastics (or even on their existence), a number of authors have come to the conclusion that the phenomenon exists and that statistical strength theory may be used to quantify it. Further, even though this theory is statistical in nature, very little statistical analyses of the results and trends are reported. In much of the literature, the authors simply fit the model to the data to obtain the appropriate parameters and then uses this to predict specific data values. This may appear, on face value, to be acceptable. However, there is a degree of scatter in the data and some indication of the confidence with which these predictions are made should be given. Hitchon and Phillips [10] do carry out some statistical significance tests but come to the conclusion that, for the small strength variations seen, the number of samples considered is too small to enable these effects to be distinguished from the experimental variability. This is also pointed out by Wisnom, [42] and some average values are based on around five or even one observation [38, 63], with no indication as to how this affects the confidence with which the predictions may be accepted.

Hitchon and Phillips [10] also highlight the fact that the statistical theories do not allow for variations in composite quality owing to factors such production-related issues. The literature appears to have lost sight of the original impetus for the study of size effects—the problem of scaling up test data to full-scale. If such effects as manufacturing/processing variables, quality and volume of production could be important, then why have they been ignored? This could be because many of the studies were concerned with high quality, aerospace type composites. The work that was not appear to have directly translated the methods already used for such materials to those with much more variable mechanical properties. In fact, Hitchon and Phillips [10] do try to explore this issue, but do so in a non-systematic way and hence are unable to interpret their results easily. It is important that such sources of variation should be investigated.

The methodology used to examine the problem should recognise the multiplicity of variables involved in composites testing (and the consequential need to examine the influence of as many of the most significant ones as possible) and the fact that even coupon testing is resource intensive. A very simple, economical yet comprehensive approach is to use statistical factorial experiments to generate the test scheme. The scheme will first need to be applied to readily comparable results in existing literature, such as flexural and tensile tests on UD coupons in both glass and carbon (see Sutherland et al. [1]). Then, after a selection of the important fabrication variables, it will be appropriate to apply the technique to hand laid-up WR laminates such as those used in shipbuilding (see Sutherland et al. [2]).

6. Closure

Dimensional analysis and the theory of models is used to explain what is meant by the term 'size effect' as applied to the properties of a structural material. The mathematical models developed to describe this effect have been presented. The work carried out thus far to try to reconcile the reality of experimental observation with the theories postulated has been reviewed. An overview of the areas, results and shortcomings of this literature is presented.

In summary, an investigation of possible composite materials strength size effects concerns both a large number of pertinent variables and experimental data subject to considerable scatter. This type of problem requires an efficient experimental programme and statistical analysis techniques in order to separately estimate the effects of each variable, and also to distinguish these effects from the random variation in the experimental data. The methods of statistically designed experimentation have been developed to benefit exactly this type of problem, hence it is advantageous to use them here and an introduction to this field is given in Sutherland et al. [1]. This new approach to the question of strength size effects for composite materials, was used in a comprehensive test program, see Sutherland et al. [1, 2]. In contrast to previous work which, in the main, dealt with high, quality pre-preg carbon/epoxy laminates for use in the aerospace industries, this study has focused on the much more variable, shipbuilding quality marine composites.

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